Repos, Bailouts, and Instability in the Shadow Banking System*

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Abstract

We study a model in which a risk-pooling intermediary such as a money market mutual fund (MMMF) is exposed to runs. In addition to providing risk-pooling services to investors, the MMMF lends funds to borrowers secured by collateral as in security repurchase transactions which are frequently used by intermediaries in the shadow banking system. Inspired by events during the financial crisis, we show that bailouts of such risk-pooling intermediaries are part of an efficient social insurance scheme in the event that a run emerges. Moreover, bailouts can help minimize the costs of "runs on repos" in which there would be large-scale liquidation of collateral as a result of the liquidity crisis. However, this observation does not imply that optimal intervention completely isolates shadow-banking intermediaries from a crisis. In fact, optimal public sector intervention imposes costs on money market funds by requiring them to liquidate some collateral. On the other hand, a commitment to no bailouts contributes to financial instability as the repo market collapses in the wake of a run without a public safety net.

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1 Introduction

The shadow banking system consists of many institutions which operate in a similar manner as traditional banks. However, in contrast to traditional banks, these non-bank financial institutions are subject to much less regulation. In turn, they are able to offer higher returns but do not have access to deposit insurance or the discount window. In particular, money market mutual funds (MMMFs) are an important group of intermediaries in the shadow banking system. As they mimic the demand deposit aspects of traditional banks, they are an attractive investment. Yet, without government backstops, they may also be susceptible to excessive withdrawals. To address this source of instability in the shadow banking system, the objective of this paper is to analyze the behavior of MMMFs facing runs in a financial crisis. We also investigate whether government intervention can prevent extreme economic losses.

MMMFs play a critical role in U.S. capital markets. Over the past 30 years, total net assets held by MMMFs have risen from roughly \$233 billion to over \$2.7 trillion.[1] Such an increase represents growth in asset holdings from 5% of GDP to over 16% of GDP. Moreover, Kacperczyk and Schnabl (2013) point out: "Money market funds are the largest provider of short-term financing to financial institutions,..., and they are also the largest provider of liquidity to corporations, issuing about the same amount of demand deposits as the entire U.S. commercial banking sector." (p. 1074).

The principal way that MMMFs extend funding is through security repurchase agreements (or 'repos'). For example, at the end of the third quarter of 2014, over \$575 billion of funding were extended through repos. In particular, the amount of repo funding by MMMFs peaked at \$600 billion in the third quarter of 2008 – the most significant point in the recent financial crisis. In fact, Gorton and Metrick (2012) characterize the financial crisis as a large-scale "run on repo."

A typical repurchase agreement starts with a lender purchasing illiquid assets (collateral) from a borrower. The purchase is considered a "true sale" in which the lender obtains full property rights to the collateral throughout the duration of the repo. Consequently, the lender may sell or trade the collateral at any time.[2] The second leg of the repo occurs when the borrower repurchases the collateral from the lender. The repurchase price (repo price) will be higher than the lender's initial purchase price. The percentage difference between the two prices reflects the interest rate on the loan or the "repo rate."

Although repos are short-term, the maturities can range anywhere from one day to three months.[3] However, any open-ended fund such as a MMMF is still characterized by a maturity mismatch. This mismatch allows for the possibility of a "bank run" synonymous with the seminal work of Diamond and Dybvig

[1] 2014 Investment Company Fact Book, published by the Investment Company Institute.

[2] For additional discussion, see Garbade (2006), International Capital Market Association (2011) and Morrison, Roe, and Sontchi (2014).

[3] The weighted average maturity of a money market fund portfolio cannot exceed 60 days as stated by the SEC in 2010.

(1983). The excessive withdraws during a run cause the fund to be undercapitalized. Notably, the Financial Stability Oversight Council (FSOC) states in a November 2012 report:

> "In the event of shareholder redemptions in excess of a MMF's available liquidity, a fund may be forced to sell less-liquid assets to meet redemptions. In times of stress, such sales may cause funds to suffer losses that must be absorbed by the fund's remaining investors, further reinforcing the first-mover advantage."

The first-mover advantage problem is of particular importance in the current structure of MMMFs. Shares of a MMMF are calculated based on the net asset value (NAV) of the fund's portfolio. Therefore, as the value of the fund's portfolio diminishes so do the value of its shares. Subsequently, investors who withdraw their funds before the portfolio value falls will suffer no losses. This creates an environment where investors will withdraw their funds en masse when losses appear possible. MMMFs will then quickly seek out liquidity to meet the unexpectedly high demand for redemptions. In fact, Begalle, Martin, McAndrews, and McLaughlin (2013) argue that MMMFs face strong incentives to sell assets immediately, even at greatly reduced prices, during an impending crisis. As we explain below, events during the recent financial crisis have made this dramatically clear.

In only four days following the bankruptcy of Lehman Brothers, more than $300 billion 'ran' from money market funds. In order to limit the damage and promote the stability of MMMFs, the U.S. Treasury instituted a temporary guarantee program on money market fund investor shares. The guarantee program essentially provided unlimited deposit insurance for the 38,000 shareholder accounts worth around $3.8 trillion.[4] Although the Treasury paid out no claims during the twelve months it was in place, lawmakers have since installed several barriers against the renewal of such a program. Specifically, Title I of the Dodd-Frank Act established the Financial Stability Oversight Council which requires the council to eliminate expectations on the part of shareholders, creditors, and counterparties of non-bank financial companies that the government will shield them from losses in the event of failure.

Ideally, policymakers would like for MMMFs to self-insure against runs – as evident by the recent increases in capital requirements. However, holding more capital cannot insure against large scale financial crises like that of 2007-2008. As Diamond and Dybvig (1983) demonstrate, bank runs are possible whenever an institution holds long-maturity assets and issues short-maturity liabilities.[5] Therefore, it is important that policymakers understand how intermediation breaks down in the shadow banking system during times of crisis. To understand this behavior, it is important to first construct a process that resembles the type of borrowing and lending occurring in the shadow banking sector. Consequently,

[4] Information provided by the Investment Company Institute 2014 Factbook.

[5] Money market funds are required to hold a minimum of 10 percent of their assets in cash daily.

our framework is designed to mimic that of a MMMF engaging in repo markets in the presence of self-fulfilling investor runs. However, the structure can be generalized to any "open-end" fund that participates in repo lending.[6]

Our framework builds on the work of Diamond and Dybvig (1983) and Keister (2014). Notably, the framework studies an environment in which investors are exposed to liquidity risk and there is limited commitment on the part of financial market participants. Fiscal policy is modeled as a decision to fund public goods with lump-sum taxes. In contrast to Diamond and Dybvig and Keister, investment in our model is not reversible as the intermediary must decide how much to lend to borrowers through security repurchase agreements. As previously noted, MMMFs represent one of the largest sources of funding in repo markets.[7] Thus, the risk-pooling intermediary should be viewed as a participant in the shadow banking system in contrast to standard Diamond-Dybvig intermediaries who directly have access to investment returns. Consequently, the investment decision of the intermediaries in our framework is non-trivial compared to standard models of risk-pooling institutions.

Moreover, intermediaries cannot easily extract funds from the repo market when additional liquidity is needed. Instead, collateral will be utilized. In particular, Gorton and Metrick (2012) note that collateral is the most important part of a repo. To begin, the repo price on collateral is analogous to the interest rate on loans, and collateral itself serves as deposit insurance. The extent of over-collateralization (also interpreted at the 'haircut') is similar to a reserve ratio in the traditional banking system – a feature of the repurchase market that, like a reserve requirement, can be used to increase market stability. In addition, collateral also serves as an incentive device for borrowers to engage in the repurchase. With all this, in the event of large-scale runs in the shadow banking system, collateral clearly plays a critical role. Yet, in such an environment, would public sector intervention be appropriate to promote stability among MMMFs and the repo market?

Inspired by events during the financial crisis, we show that bailouts of such risk-pooling intermediaries are part of an efficient social insurance scheme when a run emerges. In a decentralized setting, income taxes which fund public goods are inefficiently high. As a result, deposit funding in the financial system will be low, causing funding offered to borrowers to be inefficient and repo rates to be too high. However, in the unlikely event that a run emerges, the high level of public resources can be used to minimize the costs of "runs on repos" in which there would be large-scale liquidation of collateral as a result of the liquidity crisis. Nevertheless, this observation does not imply that optimal intervention completely isolates shadow-banking intermediaries from a crisis. In fact, optimal public sector intervention imposes costs on money market funds by requiring them to liquidate some collateral. On the other hand, a commitment to no bailouts contributes to financial instability as the repo market collapses in the wake of a run without a public safety net.

[6] A open-ended fund will buy back shares that its investors wish to sell at any time.

[7] MMMFs rank second in total assets held in the repurchase market. The largest category of lenders in repos are brokers and dealers.

The remainder of the paper is as follows. Section 2 outlines the environment for the model. Section 3 studies investment decisions and repo activity in a Planner's allocation. Next, section 4 considers a setting where fiscal authorities have discretion to bailout intermediaries in the event that a run by investors occurs. Section 5 compares outcomes under discretion to an environment where fiscal authorities are committed against public-sector bailouts of private institutions. Section 6 provides concluding remarks. Proofs of major results are provided in the Appendix.

2 The Model

The model extends Keister (2014) to include a money market where risk-pooling intermediaries engage in security repurchase transactions with borrowers as in standard repo agreements between a MMMF and another intermediary such as an investment bank or a broker-dealer.[8] Similar to Diamond and Dybvig (1983) and Keister (2014), investors are exposed to idiosyncratic liquidity risk and seek access to the risk-pooling services provided by MMMFs. In contrast, borrowers seek access to funding that can be obtained from such intermediaries. As in Cipriani et al. (2014) and Allen and Gale (2000), we study the effects of unanticipated shocks whereby investors run on MMMFs. This reflects behavior experienced by MMMFs following the bankruptcy of Lehman Brothers which led to a collapse of repo markets.

2.1 Preferences and Technologies

There are two different types of agents in our model. The first is a typical depositor (henceforth called an "investor") who invests their endowment into a standard *risk-pooling intermediary* (or intermediary for short). This intermediary insures investors against liquidity risk as in Diamond and Dybvig (1983). The second agent is a "borrower". *Borrowers* have access to an investment technology but do not have any resources to operate the technology. However, they do possess a collateral good that can be sold to the risk-pooling intermediary with the intention of buying it back at a later date. In this manner, the risk-pooling intermediary and borrowers take part in a repurchase transaction which is a common financing arrangement among participants in the shadow banking sector.[9]

The economy lasts for three periods, $t = 0, 1, 2$. In the economy, there is a population mass 1 of investors indexed by $i \in [0, 1]$. Investors value consumption

[8] A broker-dealer is in the business of buying and selling securities. For example, a broker-dealer may be a subsidiary of a large commercial bank. Alternatively, a broker-dealer may be a subsidiary of an investment bank which is a modern, market-based intermediary in the shadow banking system rather than a traditional deposit-based intermediary.

[9] Boulware, Ma, and Reed (2014) study the impact of monetary policy shocks on repo activity by the primary dealers of the Federal Reserve System which are largely composed of investment banks.

of a private good and access to the economy's public good as follows:

$$U^I(c_1, c_2, g; \omega_i) = u(c_1 + \omega_i c_2) + u(g),$$

where u is a constant relative risk aversion utility function with coefficient γ.[10]

The indicator variable ω_i indicates whether the investor is *impatient* or *patient.* That is, its value determines whether the investor desires consumption in period 1 or period 2. The value of ω_i is random with support $\Omega_i \equiv \{0, 1\}$ and π is the probability of being impatient. By the law of large numbers, π also represents the total fraction of investors who are impatient but the realization of the individual's preferences is private information. If $\omega_i = 0$, the investor is impatient, and they are patient if otherwise.

In contrast to investors, borrowers do not face any idiosyncratic risk and they do not derive utility from access to the economy's public good. In particular, a borrower only values consumption of the private good in period 2. The population mass of the borrowers is equal to 1 and their preferences are given by:

$$U^B(b_2) = u(b_2) = \frac{(b_2)^{1-\gamma}}{1-\gamma}$$

Each investor is endowed with 1 unit of the private good in period 0. Each borrower is endowed with some amount z of the collateral good in period 0. Private production in the economy primarily occurs through the borrowers' investment technology which can transform units of the private good in period 0 into $R > 1$ units of the private good in period 2. The risk-pooling intermediaries only have access to a storage technology in which one unit of the private good in period 0 yields one unit of the private good in the following period.

Two other technologies also exist. One technology, held by the fiscal authority, transforms units of the private good one-for-one into units of the public good in any period. The other technology, held by both the financial intermediary and the borrower, can transform units of the collateral good into units of the private good in periods 1 and 2. However, the transformation rate varies between the market participants. Notably, the rate at which the intermediary can transform collateral is equal to p_F while the borrowers' transformation rate is $p_B \geq p_F$. As described by Gorton and Metrick (2012), broker-dealers often make markets in securities that they use as collateral. On the other hand, money market mutual funds are not well-positioned to sell securities that they hold as collateral. Therefore, the assumption that $p_B \geq p_F$ reflects the different valuations of collateral across market participants.

[10] Keister (2014) allows the utility function from consumption of the private good to differ from the utility from access to the economy's public good. However, as we explicitly model the repo market, we have additional variables to determine including the repo rate in each aggregate state and the investment by the intermediary before the realization of the aggregate state. We also endogenously determine the amount of collateral liquidation which can occur under each aggregate state. With the presence of five additional endogenous variables in our model, it is important that investors have the same utility function for both types of goods.

Withdrawals are executed following a sequential-service constraint. This concept allows the investor to observe their position in the withdrawal order and make their consumption decision accordingly. Investors, based upon their "position in line" i, choose when to withdraw funds and will receive payments by the financial intermediary contingent on the number of withdrawals that have already taken place. One might interpret that the earliest investors are perhaps the most savvy among all the investors. For example, investors with a small value of i could be interpreted as "institutional" investors and others as "retail" investors.

2.2 Unexpected Shocks and Financial Instability

Crises in the model are based on an unexpected shock regarding the "state of the world".[11] There are two possible states. Let $S = (\alpha, \beta)$ be the set of extrinsic signals representing the good and bad states, respectively. The good state relates to normal investor sentiment regarding the degree of early liquidation to be experienced by MMMFs. However, the bad state signals an impending financial crisis where all individuals will seek to withdraw funds in period 1 regardless of their individual realizations of their discount rates.

That is, investors construct their withdraw profile:

$$y_i : \Omega \times S \to \{0, 1\} \ ,$$

where y_i corresponds to withdrawing early or waiting until period 2. The realization of a financial crisis causes fragility in the system when some proportion of patient investors decide to withdraw their funds early:

$$y_i\left(1, \beta\right) = 0 \tag{1}$$

As we will show, a crisis across MMMFs can have negative consequences for the repo market:[12]

Definition 1: *Instability in the repo market occurs when participation in the money market (at any point in time) is no longer incentive compatible for either market participant.*

[11] According to the International Capital Market Association, about half of all repurchase agreements have terms of one month or less. Given the short length of these agreements and the low probability of a financial crisis from historical data, we consider crises to be unexpected shocks in our environment. For example, Hsiang and Jina (2014) cite data from Reinhart and Rogoff (2009) which indicates that the historical probability of a financial crisis is less than 0.1%. This shock is modeled in a manner similar to Cipriani et. al. (2014) and Allen and Gale (2000).

[12] Rosengren (2014) recognizes the potential for poor behavior to spill across to numerous financial markets: "Financial instability occurs when problems (or concerns about potential problems) within institutions, markets, payments systems, or the financial system in general significantly impair the supply of credit intermediation services – so as to substantially impact the expected path of real economic activity."

In contrast to Keister (2014), the actions of the investors who deposit funds at the risk-pooling intermediary have repercussions that extend beyond the intermediary. Since our model contains a money market, we are able to make distinct statements regarding the financial instability of money markets created by investor runs. Notably, we will demonstrate that the repo market breaks down as MMMFs experiencing unanticipated demand for liquidity will sell the collateral acquired in the initial stage of the repo. For example, there were widespread "fire sales" of assets shed by MMMFs after the bankruptcy of Lehman Brothers in Fall 2008. Consequently, in the absence of a policy response, repo rates can increase significantly and credit intermediation breaks down.

3 The Planner's Allocation

In order to draw direct comparisons to Keister (2014), we focus on the actions of a narrow-minded Planner who seeks to maximize aggregate expected utility across investors. This reflects recent discussions since the crisis which have focused on providing recommendations to improve the stability of MMMFs. Towards that end, we want to directly show how efficient allocations (from the perspective of the funds) would be determined. Moreover, modeling the repo market provides important insights regarding instability in the shadow banking system. The ex-ante aggregate utility of investors is:

$$\int_0^1 \left[U^I\left(c_1\left(i\right), c_2\left(i\right), \tau; \omega_i\right)\right] di$$

The timeline of moves and events is described as follows:

t=0.1 Investors' endowments are taxed and the remainder is deposited with the risk-pooling intermediaries.

t=0.2 The Planner chooses portfolios to maximize expected utility across investors.

t=0.3 The initial stage of repurchase agreements occurs where intermediaries and borrowers exchange private goods and collateral goods in a competitive market.

t=1.0 Investors with $i \leq \theta$ decide to withdraw funds.

The aggregate state is realized by the financial intermediary and fiscal authority.

t=1.1 Funding decisions regarding the remaining impatient and patient investors are made.

t=2.0 Borrowers choose how much collateral to repurchase.

t=2.1 All remaining investors withdraw their funds.

Within the timing of events listed, there are three unique stages. Importantly, at *t=0.3*, the initial leg of repurchase transactions occurs where intermediaries transfer units of the private good to borrowers in exchange for the collateral good. In order to avoid the potential for strategic default by the borrower, the intermediary requires that the funds lent must be secured by collateral. Moreover, repo agreements generally involve some degree of overcollateralization or a "haircut".[13,14] For each unit of funding provided, the borrower must transfer δ units of the collateral good to the intermediary. Notably, $\delta > 1$ represents the level of overcollateralization. Consequently, the total amount of collateral received is $x\delta$ where x is the amount of investment in the repo market by the risk-pooling intermediaries.

Second, after the aggregate state has been revealed, the intermediary will make a decision regarding collateral liquidation. Specifically, at *t=1.1*, intermediaries will liquidate some portion (λ_s) of their collateral if they find it optimal to distribute a greater amount of funding than they currently have on hand to impatient investors. Collateral is liquidated at rate p_F using the transformation technology discussed earlier. In this manner, investor runs can trigger instability in the repo market as borrowers would not be able to repurchase all of the collateral that they transferred to the intermediary.

The third important stage further highlights the potential for liquidity crises to spillover to the repo market. At *t=2.0*, borrowers meet up with financial intermediaries and determine the price necessary to repurchase the collateral – thus completing the final leg of the repurchase agreement. After a liquidity disruption, the repo price will increase in the absence of a policy response. In fact, the price might be so high that it is no longer incentive compatible for borrowers to repurchase their collateral. In turn, strategic default by the borrower may take place.

We will now solve for the Planner's allocation in the model. It is important to note that we only want to study subgame perfect equilibrium outcomes. Therefore, we proceed using a backwards-induction approach. Since there is a strategic game between the investors and the Planner, we will consider the following 'partial run' strategy profile of investors:

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[13] For general information about haircuts, see Gorton and Metrick (2012). Additional work regarding their use can be found in Dang, Gorton, and Holmström (2013) and Otto and Reed (2014).

[14] To determine the "haircut" simply use the formula $1 - \frac{1}{\delta}$. The definition of a haircut in this way is the same used in Dang, Gorton, and Holmström (2013).

$$
\begin{aligned}
y_i(\omega_i, \alpha) &= \omega_i \qquad \forall i \\
y_i(\omega_i, \beta) &= \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{ for } \left\{ \begin{array}{c} i \leq \theta \\ i > \theta \end{array} \right\}
\end{aligned} \tag{2}
$$

Under this withdrawal strategy, patient investors in state α will always wait to withdraw. On the other hand, *some* patient investors in state β who have the opportunity to withdraw before the planner realizes the state of the world will choose to withdraw early. Hence, only a segment of the population will run in state β.

3.1 Period 2: Settlement (Repurchase of Collateral)

Before the intermediary distributes the receipts from the repo market to the remaining patient agents, it first meets up with the borrower to complete the final leg of the repo transaction. Note that depending on the state of the world, the intermediary liquidated a fraction, λ_s, of the collateral amount $x\delta$. Therefore, at this stage, the intermediary possesses $(1-\lambda_s)x\delta$ (which is constrained to be less than z) units of the collateral good in state s. As a result, this is also the amount of collateral available for repurchase. It remains to be determined how much the borrower will pay to re-acquire the remaining amount of collateral.

The repo price, r_s, is the state-contingent price of repurchasing collateral that was initially used to secure funding. As mentioned, the collateral available for repurchase depends on the state of the world. Thus, income raised by the intermediary in the repo market would equal $r_s(1-\lambda_s)x\delta$. The funds raised by selling the collateral available in state s back to the dealer will be distributed to the remaining investors who seek to redeem their funds in state s.

The number of remaining patient investors, $(1-\theta)(1-\widehat{\pi}_s)$, would then obtain

$$
c_{2s} = \frac{r_s(1-\lambda_s)x\delta}{(1-\theta)(1-\widehat{\pi}_s)}
$$

in income from the intermediary.[15] In return, income for the borrowers in the repurchase would be:

$$
b_{2s} = R \cdot x - r_s(1-\lambda_s)x\delta + p_B\left(z - \lambda_s \delta x\right)
$$

As a tractable alternative to Nash bargaining, we consider that the repo price paid in state s is chosen to maximize aggregate welfare across patient investors and borrowers. Optimal pricing in this manner insures that our results concerning fragility will not be driven by inefficiencies from the price determination mechanism.

[15] The values of $\widehat{\pi}_s$ generated from the strategy profiles in (2) are represented as $\widehat{\pi}_\alpha = \frac{\pi-\theta}{1-\theta}$ and $\widehat{\pi}_\beta = \pi$. The value of $\widehat{\pi}_s$ therefore denotes the state-dependent fraction of investors who withdraw early.

The determination of the repo price is as follows. The total measure of individuals who wait until period 2 in state s to redeem their funds is equal to $(1-\theta)(1-\widehat{\pi}_s)$. The population of borrowers seeking to repurchase their collateral is equal to 1. The repo price depends on the total number of investors redeeming funds in period 2 and borrowers' utility:

$$\max_{r_s}(1-\theta)(1-\widehat{\pi}_s)u\left(c_{2s}\right)+u\left(b_{2s}\right),$$

Proposition 1. *The repo price is:*

$$r_s(\lambda_s,x,\delta)=\left(\frac{(1-\theta)(1-\widehat{\pi}_s)}{1+(1-\theta)(1-\widehat{\pi}_s)}\right)\frac{\left[R\cdot x+p_B\left(z-\lambda_s x\delta\right)\right]}{(1-\lambda_s)x\delta} \tag{3}$$

In turn, the repo rate is $r_s(\lambda_s,x,\delta)\delta-1$.

The social returns from completing the second leg of the repo are equal to $\frac{R\cdot x+p_B(z-\lambda_s x\delta)}{x}$. However, given the total units of collateral available for repurchase, the repo price depends on the proportion of remaining investors who are patient as a fraction of the total size of the repo market in period 2. That is, the repo price is higher if there are more patient investors to be funded by the investment returns from the repo market.

Corollary 1: *Based upon the interest rates in the repo market, the state-dependent amount of consumption among the borrowers and patient investors is:*

$$c_{2s}(\lambda_s,x,\delta)=\frac{\left[R\cdot x+p_B\left(z-\lambda_s\delta x\right)\right]}{1+(1-\theta)(1-\widehat{\pi}_s)}=b_{2s}(\lambda_s,x,\delta)$$

From the perspective of the patient investor, consumption is just the amount of income earned by borrowers dispersed among the size of the repo market in period 2. As is easily observed, however, the extent of collateral liquidation adversely affects consumption among patient investors. Therefore, Corollary 1 implies that the planner must carefully consider their decision whether to liquidate collateral in period 1.

Consumption for the borrowers also decreases with collateral liquidation. More importantly, borrowers in this environment with limited commitment are not forced to accept the repo rate that is proposed. Notably, the incentive compatibility constraint of borrowers in the settlement stage of a repurchase transaction is:

$$\left(1-\lambda_s\right)p_B\delta x\geq\left(\frac{(1-\theta)(1-\widehat{\pi}_s)}{1+(1-\theta)(1-\widehat{\pi}_s)}\right)\left[R\cdot x+p_B\left(z-\delta x\right)\right] \tag{4}$$

In other words, the amount of consumption gained through repurchasing the collateral must be greater than the proportion of private goods forfeited by walking away and strategically defaulting. Alternatively, borrowers' incentives can be expressed in terms of the amount of funding obtained from the intermediary:

$$x \geq \frac{(1-\theta)(1-\widehat{\pi}_s)p_B z}{p_B \delta \left[\left(1+(1-\theta)(1-\widehat{\pi}_s)\right)(1-\lambda_s)+(1-\theta)(1-\widehat{\pi}_s) \right] - R(1-\theta)(1-\widehat{\pi}_s)} \tag{5}$$

Higher investment translates into more collateral which in turn increases the incentive for borrowers to repurchase. From equation (5), it is also clear that liquidation of collateral by the risk-pooling intermediary tightens the participation constraint.

3.2 Period 1: Collateral Liquidation and Bailout Funding

Prior to the realization of the aggregate state, each intermediary would have committed to providing returns which offer c_1 to all investors who show up in period 1 to withdraw their funds. After $\theta \in [0, \pi]$ investors have withdrawn funds, both the intermediary and the fiscal authority realize the aggregate state of the economy. If $\theta = \pi$, the intermediary would only learn that the aggregate economy is in crisis after all of the anticipated demand for redemptions (which should only have occurred by those truly impatient) were realized. This is the standard approach in Diamond-Dybvig models of fragility.

However, if $\theta < \pi$, the intermediary has a chance to revise payments prior to realizing the total anticipated demand for redemptions by depositors. That is, the Planner would have the ability to adjust payments to impatient investors (along with those who are privately patient) prior to the anticipated demand for short-term redemptions that should have taken place.

After the state of the world has been revealed, the Planner solves the following problem of maximizing aggregate utility among all of the remaining $(1-\theta)$ investors (by construction, c_1 has already been allocated to each of the θ individuals who showed up at the intermediary):

$$V(\psi_s; \widehat{\pi}_s) \equiv \max_{(c_{1s}, c_{2s})} (1-\theta)\left(\widehat{\pi}_s u(c_{1s}) + (1-\widehat{\pi}_s) u(c_{2s})\right)$$

To determine the state-contingent resource constraint facing the Planner, first note that there are a total of $(1-\theta)$ individuals seeking redemptions once the aggregate state has been revealed. Of that group, $(1-\widehat{\pi}_s)$ will show up in period 2. Therefore, the total consumption among the patient individuals awarded ex-post will be: $(1-\theta)(1-\widehat{\pi}_s)c_{2s}$. By comparison, $(1-\theta)\widehat{\pi}_s c_{1s}$ would be provided to the impatient investors in state s.

Each intermediary initially had total funding in the amount $1-\tau$ where τ represents the tax imposed in period 0. The resources available to the intermediaries in period 1 are also contingent upon x (funds were lent to borrowers in the repo market in period 0) and θc_1 (total redemptions provided before the realization of the aggregate state).

We begin our analysis of period 1 choices with those occurring in state α.

3.2.1 Optimizing Consumption in the Good Aggregate State

In the good state, each intermediary solves the following problem:

$$\begin{aligned} V(\psi_\alpha;\widehat{\pi}_\alpha) \quad \equiv \quad & \max_{c_{1\alpha},\,\lambda_\alpha} (1-\theta)\left(\widehat{\pi}_\alpha u(c_{1\alpha}) + (1-\widehat{\pi}_\alpha)\, u\left(\frac{[R\cdot x + p_B(z-\lambda_\alpha\delta x)]}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\right) + \\ & \mu_\alpha\left[p_F\lambda_\alpha\delta x + 1 - \tau - x - \theta c_1 - (1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha}\right] \end{aligned}$$

In addition to showing the implications of a widespread liquidity crisis for risk-pooling intermediation, our model is also designed to understand the determination of repo funding and credit intermediation in the shadow banking sector. In the absence of a financial crisis, lenders enter the repurchase stage with all of the collateral purchased in the initial leg of the repo agreement:

Proposition 2:. *Intermediaries will not liquidate any of the collateral in the good state. That is,* $\lambda_\alpha = 0$ *iff*

$$\frac{u'(c_{1\alpha})}{u'(c_{2\alpha})} < \frac{p_B}{p_F}\left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right) \tag{6}$$

The condition on liquidation of collateral can be written as a condition on the marginal rate of substitution between early and late consumers. As long as the marginal rate of substitution is not too high – that is, the desire of the planner to allocate additional income to the impatient consumers is not too high – then the Planner would not choose to liquidate any of the collateral. This is more likely to hold if the relative collateral value between the borrower and the intermediary is higher. In other words, as the economic inefficiency of collateral liquidation becomes greater. In addition, the condition is more difficult to achieve as the information asymmetry between investors and the Planner grows (ie. θ increases).

In state α, bailouts are also unnecessary. Therefore, the amount of consumption across investors in the good state is:

$$c_{1\alpha}(c_1, x, \tau) = \frac{1-\tau-x-\theta c_1}{(1-\theta)\,\widehat{\pi}_\alpha} \tag{7}$$

$$c_{2\alpha}(c_1, x, \tau) = \frac{[R\cdot x + p_B z]}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \tag{8}$$

Substituting the period 1 solutions for $c_{1\alpha}(c_1, x, \tau)$ and $c_{2\alpha}(c_1, x, \tau)$ shows that the condition in Proposition 2 can be expressed in terms of the extent of investment in the repo market:

Corollary 2: *Intermediaries do not liquidate collateral in the good state iff:*

$$x < \frac{1 - \tau - \theta c_1 - p_B z \left(\frac{(1-\theta)\widehat{\pi}_\alpha}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right) \left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\alpha)}{(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}}{\left[1 + R\left(\frac{(1-\theta)\widehat{\pi}_\alpha}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\alpha)}{(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]} \tag{9}$$

Corollary 2 reevaluates intermediaries incentives' to liquidate collateral as an upper bound on investment. This upper bound highlights an important feature of the model. When intermediaries invest at lower levels, they have minimal access to collateral to liquidate. With fewer resources to liquidate, the marginal value of each unit of collateral increases further strengthening the condition in (6).

3.2.2 Optimizing Consumption in the Bad Aggregate State

In the event that a financial crisis is underway, given all investors are following (2), the additional withdrawals from the run causes consumption for remaining investors with $i > \theta$ to decline. To maximize aggregate utility across the remaining investors, the Planner can inject capital back into the intermediaries by liquidating some of its collateral holdings. It may also choose to transfer an amount of resources to intermediaries, k, by diverting funding away from the public good so that the intermediary would not have to liquidate as much collateral to meet the demand for redemptions.

When considering the decisions of financial intermediaries in the bad state, it is helpful to first look at the resource constraint facing the Planner:

$$\frac{Rx + p_B z - [1 + (1-\theta)(1-\widehat{\pi}_\beta)]\, c_{2\beta}}{p_B \delta x} = \lambda_\beta$$

If the Planner intends to give more consumption to patient investors, then he cannot liquidate as much collateral. However, for a given amount of $c_{2\beta}$, the planner can allocate more of the collateral for liquidation if the returns from the borrowers' investment projects are higher (and thus the repo price is higher).

By comparison, looking at the constraint on impatient investors:

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

The proportion of collateral that must be liquidated is lower if there is more income available after funding of the public good, investment, and payments to the earliest withdrawers. In contrast to the resource constraint for state α, we now take into account the possibility of a bailout – denoted by $k \geq 0$. As we can see, choosing $c_{1\beta}$ and $c_{2\beta}$ pins down λ_β.

Constructing the resource constraint through the collateral liquidation variable (λ_β) yields:

$$\psi_\beta = \frac{p_F}{p_B} Rx + p_F z + 1 - \tau + k - x - \theta c_1,$$

Thus, the feasibility constraint for our intermediary can be described by:

$$(1-\theta)\widehat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B}\left(1 + (1-\theta)(1-\widehat{\pi}_\beta)\right) c_{2\beta} = \psi_\beta$$

Therefore, the Planner's objective once the aggregate state has been revealed is equal to:

$$\begin{aligned} V\left(\psi_\beta; \widehat{\pi}_\beta\right) \quad \equiv \quad & \max_{c_{1\beta},\, c_{2\beta}} (1-\theta)\left(\widehat{\pi}_\beta u\left(c_{1\beta}\right) + \left(1-\widehat{\pi}_\beta\right) u\left(c_{2\beta}\right)\right) + \\ & \mu_\beta \left[\psi_\beta - \left[(1-\theta)\widehat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B}\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right] c_{2\beta}\right]\right] \end{aligned}$$

Proposition 3: *In the event of a crisis the financial intermediary liquidates some proportion of collateral to satisfy:*

$$\frac{u'\left(c_{1\beta}\right)}{u'\left(c_{2\beta}\right)} = \frac{p_B}{p_F} \frac{(1-\theta)\left(1-\widehat{\pi}_\beta\right)}{\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right]} \tag{10}$$

For intermediaries to liquidate collateral in an effort to satisfy (10) then it must be the case that intermediaries realize ex-post an over-investment of investor resources into the borrower's investment technology.

Because we construct the resource constraint through λ_β, we place constraints on the intermediary's liquidation value in order to produce reasonable solutions regarding λ_β. The following corollary ensures that intermediaries cannot acquire more or liquidate more collateral than they hold:

Corollary 3: *Suppose that p_F satisfies $\underline{p_F} < p_F < \overline{p_F}$. Under this condition, $\lambda_\beta \in (0, 1)$.*

The lower bound in Corollary 3 implies that the collateral must have sufficient liquidation value in order to sell it prematurely instead of waiting for the second leg of the repo to be completed. The upper bound implies that the collateral must not be too valuable – otherwise, the lender would have an incentive to sell all of it instead of raising funds in the repo market.[16]

Knowing the values of $\widehat{\pi}_s$ generated by 2, we are able to state:

Corollary 4: *The MRS in the bad state is lower than the upper bound in the good state.*

Hence, the lower marginal rate of substitution in the bad aggregate state – caused by the moral hazard problem among investors – leads intermediaries to liquidate collateral in the bad aggregate state. To see the role of the asymmetric information, note that the marginal rate of substitution in the bad aggregate state is the same in the good aggregate state if $\theta = 0$. In comparison to Keister (2014), the total size of the financial system has a population mass of 2 since there are two groups of agents that consume: borrowers and investors.

In the event of a financial crisis, the Planner may find it in the best interest of the investors to hold back on provision of the public good and instead redistribute some of its tax revenues to promote social welfare. Thus, the Planner solves:

$$\underset{(\tau-k)}{Max}\left[V\left(p_F\lambda_\beta(x,\tau,k,c_1)\delta x+1-\tau+k-x-\theta c_1;\widehat{\pi}_\beta\right)+u(\tau-k)\right]$$

so that the size of the bailout satisfies:

$$u'(\tau-k)=\mu_\beta.$$

3.2.3 Period 0: The Planner's Investment in the Repo Market, Short-Term Returns, and Taxes

As in Allen and Gale (2000) and Cipriani, Martin, McCabe, and Parigi (2014), we consider the bad state to be an event which is highly unlikely to occur. Therefore, any realization of the bad aggregate state is unexpected causing the Planner to solve the following ex-ante problem:

$$\underset{x,\ c_1,\ \tau}{Max}\ \theta u(c_1)+\left[V\left(\psi_\alpha;\widehat{\pi}_\alpha\right)+u(\tau)\right]$$

[16] Corollary 3 is consistant with the incentive issues of a repurchase agreement that other work, such as Roe et al. (2014), has addressed.

The Planner's Early Consumption Decision

$$u'(c_1) = \mu_\alpha$$

The Planner's Investment in the Repo Market

$$u'\left(c_{1\alpha}\right) = u'\left(c_{2\alpha}\right) R \left[\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)\left(1-\widehat{\pi}_\alpha\right)}\right]$$

The term on the right-hand side represents the increase in utility among the patient investors in the good state from a higher amount of lending by intermediaries. The term on the left-hand side represents the decrease in utility among impatient agents in the good state.

As we can see above, only the consumption levels in the good aggregate state are considered when making the investment decision. This hold true because any shock regarding the aggregate state is unexpected at date 0 and, therefore, does not affect the optimal risk-sharing contract offered by the MMMF:

$$\frac{u'\left(c_{1\alpha}\right)}{u'\left(c_{2\alpha}\right)} = R \left[\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)\left(1-\widehat{\pi}_\alpha\right)}\right] \tag{11}$$

The MRS above comes from the maximization of investment in period 0. This MRS looks very similar to that provided by the state α FOC from consumption choice in period 1. However, here we can see that the right-hand side is pre-multiplied by the strength of the investment technology, R, instead of $\frac{p_B}{p_F}$. Corollary 5 (below) provides conditions where any investment solution which satisfies (11) remains consistent with the corner solution found from the period 1 optimization.

Corollary 5. *Let R be such that $\frac{p_F}{p_B}R < 1$. Then, any investment solution which satisfies (11) remains consistent with complete settlement of repos in the good state.*

Although the Social Planner invested amount x of the private good in period 0, a partial value of the investment may still be redeemed in period 1 through collateral liquidation.

The Planner's Tax Decision

$$u'(\tau) = \mu_\alpha$$

The Planner chooses a tax that evens consumption out amongst all impatient consumers. It also takes into account the optimal trade-offs between allocating income for consumption and the public good:

$$u'(\tau) = u'(c_{1\alpha}) = u'(c_1).$$

We conclude with the following statement:

Proposition 4. *Suppose that* $\delta > \max(\underline{\delta_\alpha^*}, \underline{\delta_\beta^*})$. *Further,* p_F *satisfies the conditions in Corollary 3 and Corollary 5. Then, an allocation in the planner-run financial system exists in which complete settlement of repo financing occurs in the good state. In the bad state, only partial settlement occurs.*

3.3 Fragility

The financial system is fragile if there exists an equilibrium strategy profile with $y_i(1, \beta) = 0$ *for a positive measure of investors.* As Keister (2014) explains: "Fragility thus captures the idea that the financial system is potentially susceptible to a run based on shifting investor sentiment" (p. 7). In particular, we will consider the partial-run strategy profile outlined by (2).

The strategy profile of (2) is a partial run because only some individuals would run in the bad aggregate state. Since the intermediary follows a sequential service strategy, as soon as it realizes the aggregate state is β, it will adjust interest payments to the remaining investors.[17] This adjustment to interest payments is performed by the intermediary through a decision on collateral – more specifically, λ_β. Investors will not run if they are after the θth individual since the information asymmetry between the investors and the fund would no longer be present. According to (2), investors with $i \leq \theta$, will choose to withdraw early in aggregate state β regardless of their patience or impatience.

Let y^* represent the partial run strategy profile in (2). Furthermore, y^* is part of an equilibrium if $c_1^* \geq c_{2\beta}^*$. It is straightforward to show that there exist parameter values such that this condition is satisfied. However, we need to verify whether it's also a necessary condition. That is, suppose a run is occurring in state β. Does a run lead to the conclusion that:

$$c_1^* \geq c_{2\beta}^*$$

Note the fraction of the first θ withdraws in state s is such that $\varepsilon_s \in [0, 1-\pi]$. In other words, in state β, the number of investors who 'run' is bounded by the number who do not experience a shock to preferences:

$$\varepsilon_\alpha = 0; \quad \varepsilon_\beta = 1 - \pi \tag{12}$$

[17] Keister (2014) discusses numerous examples where financial intermediaries have adjusted their liabilities during banking crises. In a similar manner, BNP Paribas suspended redemptions of three of its funds in August 2007. Upon retrospection, this event foreshadowed the coming global financial crisis.

The notation representing the remaining impatient investors where $\widehat{\varepsilon}_s$ is defined by the bounds in (12) is $\widehat{\pi}_s$.

We intend to show that if $\varepsilon_\beta > 0$, it must be the case that $c_1^* \geq c_{2\beta}^*$ holds. Otherwise, the partial run strategy outlined above does not necessarily exist in equilibrium. If this is the case then some other strategy may dominate in equilibrium. The notation

$$\widetilde{\pi}_s = \widetilde{\pi}_s\left(\widetilde{\varepsilon}_s\right)$$

will be used to represent the remaining impatient investors after the state has been revealed where $\widetilde{\varepsilon}_s$ is not defined by (12).

Let $\widetilde{y}$ be any strategy profile where the number of withdraws from investors whose $i \leq \theta$ leads to $\widetilde{\pi}_s$. In response to the strategy profile $\widetilde{y}$, the Planner allocates $\{\widetilde{c}_1, \widetilde{c}_{1s}, \widetilde{c}_{2s}\}$ for $s = \alpha, \beta$. If there is an equilibrium in which investors follow $\widetilde{y}$, it must be the case that

$$\widetilde{c}_1 \geq \widetilde{c}_{2\beta}.$$

Due to the strategy adopted by the ε_s individuals,

$$\widehat{\pi}_s\left(\widehat{\varepsilon}_s\right) = \frac{\pi - \left(1 - \widehat{\varepsilon}_s\right)\theta}{1-\theta}$$

represents the fraction of remaining impatient investors after θ withdraws have been made. The conditional probability that someone shows up early after the first θ withdraws depends on the number of those who are truly impatient relative to the fraction of the θ withdrawals made by those who were actually patient. For example, in state β under the y^* strategy, some proportion of the first θ individuals served were actually patient investors 'running' on the intermediary. The possible fraction of patient investors who choose to run is $(1-\pi)$. Consequently,

$$\widehat{\pi}_\beta\left(\widehat{\varepsilon}_\beta\right) \equiv \frac{\pi - \left(1-(1-\pi)\right)\theta}{1-\theta} = \frac{\pi - \pi\theta}{1-\theta} = \pi.$$

After θ investors have been served the Planner offers an incentive compatible arrangement such that $c_{1\beta}$ will be below $c_{2\beta}$. Thus, the conditional probability of an investor redeeming their funds early at this point just depends on the number of remaining impatient relative to those that were already served.

Proposition 5: *A Planner-run financial system is fragile if and only if* $c_1^* \geq c_{2\beta}^*$ *holds.*

Here we show that given state β the best decision for a patient investor is to run if their order in line is $i \leq \theta$. Investors in our environment do not benefit from choosing any strategy other than y^*. Therefore, moving forward in the analysis the investors will always follow the choices outlined by (2).

3.4 Bailouts and Collateral Liquidation

After the Planner realizes the state of the world, he is able to make decisions regarding the amount of collateral to liquidate and the amount of public goods to put in place. In state α, a complete settlement of repo transactions occurs as shown by Proposition 4. In addition, all of the taxes collected will be allocated towards funding for public goods.

In the event of a crisis and investor runs, absent any decision made by the Planner, the intermediary will run out of liquidity before all of the truly impatient agents have consumed in period 1. Because the Planner has a desire to provide risk-sharing among all investors, he will utilize the collateral and tax resources at his disposal to inject additional liquidity into the intermediaries' balance sheets. Both actions come with social costs.

Collateral liquidation in our environment, and unlike Keister (2014), is not modeled as completely reversible investment funding. Liquidation comes with heavy social costs, particularly for those consuming in period 2. Transferring tax resources to the intermediaries, on the other hand, is a more efficient process. Although, providing a bailout decreases the level of public goods available for all investors. In this way, the Planner also has a desire to mitigate the losses among taxpayers who value public goods. The following Proposition shows how the Planner attempts to maximize investor welfare across both tools:

Proposition 6. *The planner chooses a combination of bailouts and collateral liquidation when the bad state occurs,* $k^* > 0$ *and* $\lambda^*_\beta > 0$.

Providing a bailout decreases the level of public goods available for both impatient and patient investors. In contrast, liquidating collateral leads to lower consumption for patient investors. Thus, the planner utilizes two tools to improve social welfare in the event of a crisis. Obviously, Proposition 6 demonstrates that the planner will implement a fiscal bailout by diverting funding from public goods. Yet, the Planner also imposes costs on intermediaries by liquidating some of the collateral held by institutions. In conjunction with Corollary 3, Proposition 6 provides conditions under which the Planner will seek to avoid huge economic losses in the repo market.

4 Equilibrium Under Discretion

We now deviate from the Planner's allocation to studying equilibrium behavior in a decentralized world. In this case, the policymaker or fiscal authority has *discretion* over the amount of funds to "bailout" the intermediary in the bad aggregate state. The total size of the bailout across all intermediaries is defined as:

$$k \equiv \int k^j d\sigma(j).$$

Let $k^j \geq 0$ denote the bailout payment by the fiscal authority to intermediary j for each investor. In addition, let $\sigma(j)$ denote the distribution of investors who have allocated funds to intermediary j. Then,

$$\max_{\{k^j\}} \equiv \int V\left(\psi_\beta^j; \widehat{\pi}_\beta\right) d\sigma(j) + u\left(\tau - k^j\right)$$

subject to the feasibility constraint:

$$\psi_\beta^j = \frac{p_F}{p_B} Rx(j) + p_F z + 1 - \tau + k^j - x^j - \theta c_1^j$$

For a given size of the bailout package k per investor, the amount of the bailout entails:

$$k^j = k + \theta\left(c_1^j - \overline{c}_1\right) \qquad \forall j$$

where $\overline{c}_1 \equiv \int c_1^j d\sigma(j)$ is the average level of c_1^j in the economy.

The logic behind this specification of the bailout is that the institution paid c_1^j to θ individuals before it became aware of the aggregate state and did not know that it should restructure its interest payments to depositors. Thus, the bailout should be designed to correct the error from the information asymmetry between the investors (who know the aggregate state) and the intermediary (unaware of the aggregate state at the beginning of the period). In turn, we have:

$$\psi_\beta^j = \frac{p_F}{p_B} Rx^j + p_F z + 1 - \tau - x^j - \theta\overline{c}_1 + k$$

in which the intermediary takes $\overline{c}_1$ as given.

In choosing k^j, the fiscal authority takes into account how the size of the bailout affects the decision making of the intermediaries. Recall from the Planner's problem that bailouts directly reduce the need for intermediaries to liquidate collateral. Consequently, the Lagrangian function of the fiscal authority is:

$$\begin{aligned}
\max_k \mathcal{L} \quad = \quad & \int V\left(\psi_\beta^j; \widehat{\pi}_\beta\right) d\sigma(j) + u(\tau - k) + \\
& \mu_\beta^j \left[\frac{p_F}{p_B} Rx^j + p_F z + 1 - \tau - x^j - \theta\overline{c}_1 + k - \left((1-\theta)\,\widehat{\pi}_\beta c_{1\beta}^j + \frac{p_F}{p_B}\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right] c_{2\beta}^j\right)\right]
\end{aligned}$$

When the aggregate state is revealed the fiscal authority chooses the size of the bailout and each intermediary solves:

$$\begin{aligned}
V\left(\psi_\beta^j; \widehat{\pi}_\beta\right) \quad \equiv \quad & \max_{c_{1\beta}^j,\, c_{2\beta}^j} (1-\theta)\left(\widehat{\pi}_\beta u\left(c_{1\beta}^j\right) + (1-\widehat{\pi}_\beta)\, u\left(c_{2\beta}^j\right)\right) \\
& + \mu_\beta^j \left[\frac{p_F}{p_B} Rx^j + p_F z + 1 - \tau - x^j - \theta\overline{c}_1 + k - \left((1-\theta)\,\widehat{\pi}_\beta c_{1\beta}^j + \frac{p_F}{p_B}\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right] c_{2\beta}^j\right)\right.
\end{aligned}$$

This yields the FOCs:

$$u'\left(c_{1\beta}^{j}\right)=\mu_{\beta}^{j}$$

$$u'\left(c_{2\beta}^{j}\right)=\frac{p_F}{p_B}\frac{\left[1+(1-\theta)(1-\widehat{\pi}_{\beta})\right]}{(1-\theta)\left(1-\widehat{\pi}_{\beta}\right)}\mu_{\beta}^{j}$$

The optimal risk-sharing condition is

$$u'\left(c_{1\beta}^{j}\right)=u'\left(c_{2\beta}^{j}\right)\frac{p_B}{p_F}\frac{(1-\theta)\left(1-\widehat{\pi}_{\beta}\right)}{\left[1+(1-\theta)(1-\widehat{\pi}_{\beta})\right]}=\mu_{\beta}^{j}$$

The results are analogous to the Planner's allocation. Based upon the behavior of each intermediary, the envelope theorem applies and therefore the bailout is pinned down by:

$$u'\left(\tau-k\right)=\mu_{\beta}^{j}$$

By the envelope theorem, there are not any indirect effects of the bailout package on the interest payments to each investor. Simply put, the fiscal authority recognizes that the social cost of the bailout derives from lower provisions of the public good to everyone. The marginal benefit $\left(\mu_{\beta}\right)$ of lower provisions is the additional resources that can be transferred to the remaining $(1-\theta)$ depositors after the system has been subjected to a run.

Given that the market for investor funds and the money market are both perfectly competitive, the optimal decisions across intermediaries and borrowers will be the same. Therefore, in order to simplify the notation moving forward, we will denote the decentralized funding decisions by all intermediaries as $\left\{c_1^D, c_{1s}^D, c_{2s}^D\right\}$ for $s=\alpha,\beta$.

4.1 Intermediaries' Decisions

The financial intermediary chooses the level of income to be provided to the earliest consumers and the level of investment, taking the level of taxes imposed by the fiscal authority (τ^D) as given:

$$\underset{x,\ c_1}{Max}\ \theta u(c_1^D)+\left[V\left(\psi_{\alpha};\widehat{\pi}_{\alpha}\right)+u(\tau^D)\right]$$

Payments to the early investors:

$$u'(c_1^D)=\mu_{\alpha}$$

Investment in the Repo Market:

$$u'\left(c_{1\alpha}^{D}\right) = u'\left(c_{2\alpha}^{D}\right) R \left[\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)\left(1-\widehat{\pi}_{\alpha}\right)}\right]$$

The decision rules adopted by the financial intermediary are the same as the Planner. However, the levels of activity will not necessarily be the same across cases as the fiscal authority's tax decision plays a major role.

4.2 Fiscal Authority's Tax Decision

The fiscal authority only has direct control over the level of taxes and provision of the public good. This decision is chosen to maximize aggregate investor expected utility as follows:

$$\underset{\tau^D}{Max}\theta u(c_1) + \left[V\left(\psi_{\alpha};\widehat{\pi}_{\alpha}\right) + u(\tau^D)\right]$$

The tax decision is:

$$u'(g_{\alpha}^{D}) = \mu_{\alpha}^{D}\left[1+\frac{dx^D}{d\tau^D}\right].$$

In turn, early consumption and investment are both functions of taxes, $c_1\left(\tau^D\right)$ and $x\left(\tau^D\right)$.

As observed in the Planner's decision, lump-sum taxes are chosen to reflect an optimal trade-off between access to public goods and the loss of consumption across investors. However, as the fiscal authority attempts to maximize aggregate expected utility across investors, it also has a desire to influence the level of risk-sharing in the economy through investment in the repo market, $dx/d\tau^D$. We show in the Appendix that $dx/d\tau^D$ is negative. Therefore, the level of taxes will be higher than the level chosen by the Planner:

$$\tau^D > \tau^*$$

The excessive level of taxation emerges here because the fiscal authority only has one tool to influence the level of risk-sharing whereas the planner can directly affect investment in the repo market. This mechanism is unique to our framework – as investment in Keister is completely reversible, the investment decision is trivial. In our model, investment in the repo market is motivated by the desire to fund patient consumers. Consequently, repo funding by the intermediary affects the level of risk-sharing across all of the fund's investors.

In addition to affecting the degree to which investors obtain risk-sharing, the level of taxes affects the ability of the fiscal authority to deal with a liquidity crisis. To show this to be the case, we inspect the proportion of tax resources to intermediaries' liquid assets after the state of the world is revealed. The proportion is defined as:

$$\psi \equiv \frac{\tau}{1 - x - \theta c_1}$$

Hereafter, we refer to ψ as *public liquidity.* When an economy has higher public liquidity, the fiscal authority holds a larger proportion of the economy's liquid assets making them more agile in the event of a crisis. Using this measurement, our results suggest that the fiscal authority in a decentralized economy is more capable of dealing with a financial crisis as opposed to the Planner's case:

$$\psi_\tau^D > \psi_\tau^*$$

In essence, taxes reduce investment which has a direct effect on each intermediary's resource constraint. The higher level of taxes encourages intermediaries to provide more funding to impatient investors at the expense of patient investors. To compensate for inequality of consumption of the private good, the fiscal authority increases taxes in order to boost the level of public good because that consumption is shared across both investor cohorts. However, if a policymaker were to commit to no bailouts, then higher taxes would adversely affect the intermediaries' ability to respond to a crisis.

As in the Planner's allocation, we can show that a fiscal authority under discretion chooses a level of bailouts but also imposes costs on intermediaries in the system by inducing them to liquidate some of the collateral that they hold:

Proposition 7. *Suppose that $\delta > \max(\underline{\delta}_\alpha^D, \underline{\delta}_\beta^D)$. Further, p_F satisfies the conditions in Corollary 3 and Corollary 5. Then, an allocation under fiscal discretion exists in which complete settlement of repo financing occurs in the good state. In the bad state, only partial settlement occurs.*

We also show that the system can be fragile under discretion:

Proposition 8. *The shadow-banking system under fiscal discretion is fragile if and only if $c_1^D \geq c_{2\beta}^D$ holds.*

4.3 Bailouts vs. No Bailouts

In contrast to a setting where the fiscal authority has discretion to provide a bailout in the event that the bad state emerges, we can also consider that the fiscal authority may be committed to not bailout institutions as recent legislation such as the Dodd-Frank Act would imply. Since the information set is the same in period 0 given a commitment to bailouts or not, the ex-ante decisions of the financial intermediaries and the fiscal authority are identical. The only way to respond to a crisis under fiscal restrictions is to liquidate collateral. In this section, we will show that between the two choices, offering bailout funding in the event of a crisis is objectively the better decision. Consumption decisions in the no bailouts case are denoted as $\left\{c_1^{NB}, c_{1s}^{NB}, c_{2s}^{NB}\right\}$ for $s = \alpha, \beta$.

Proposition 9: *Under the expectation that bailouts will not occur, the set of possible economies that lead to bank runs is strictly larger than that of a discretionary regime.*

Let Φ be the set of possible parameter values that characterize an economy that leads to bank runs when the bad state occurs. Using the same set theoretic approach applied in Keister (2014), we are able to show that $\Phi^D \subset \Phi^{NB}$. Therefore, by committing to no bailouts policymakers increase the possibility of bank runs when investors realize the bad state of the economy.

Proposition 10. *Suppose that* $\underline{\delta_\beta^D} < \delta < \underline{\delta_\beta^{NB}}$. *This means there is a subset of parameter values in* Φ^{NB} *where a run on intermediaries by investors leads to a collapse of the repo market in a no bailouts regime yet survives the run with bailout funding.*

Overcollateralization (δ) is an exogenous parameter directly linked to the repo price. In accordance with the risk sharing motive of the repurchase agreement, it holds true that the higher the haircut the lower the repo price. The Proposition shows that there is a range of δ where instability in the repo market occurs if the fiscal authority refuses to rescue intermediaries but remains stable if the fiscal authority has discretion over bailout funding. We refer to a collapse of the repo market as a setting where the second leg of repos in the system is not settled. That is, strategic default by borrowers occurs. This emerges if $r_\beta > p_B$. Proposition 10 also shows that as the level of overcollateralization approaches $\underline{\delta_\beta^{NB}}$ the possibility of a market collapse decreases. This result provides support for regulation on overcollateralization as a policy tool toward stability on repo market financing.

In a no bailouts regime, intermediaries only have one way to respond to runs by investors – they liquidate large amounts of collateral as observed during the fire sales of securities which occurred after the failure of Lehman Brothers.[18] In a statement made by the Treasury in 2009:

> "An initial fundamental shock associated with the bursting of the housing bubble and deteriorating economic conditions generated losses for leveraged investors and banks... The resulting need by investors and banks to reduce risk triggered a wide-scale deleveraging in these markets and led to fire sales. As prices declined, many traditional investors exited these markets, causing declines in market liquidity."

Our model examines such an occurrence. When investors become aware of state β, the idea of massive liquidation encourages strategic default by borrowers further reinforcing the first-mover advantage among investors with $i \leq \theta$. Consequently, credit intermediation in the repo market is much more susceptible to breaking down if there is a commitment against bailouts in place.

[18] A narrative of fire-sales during a financial crisis can be found in Schleifer and Vishny (2011).

5 Comparing Bailout Cases

We have shown that commitments against bailouts can have destabilizing effects on MMMFs and repo market activity. The possibility of such instability is precisely why the Treasury instituted their temporary guarantee program. Focusing our attention away from the most destabilizing choice, we begin comparing choices under descretion to those provided by a social planner. In this manner, we seek to understand how a Planner's willingness to tolerate instability in the shadow banking system contrasts with fiscal authorities' preferences. We also provide support for the use of taxation in order to bring greater ex-ante stability to the shadow banking system.

In the discretionary case, we showed that taxes are inefficiently high as the fiscal authority attempts to influence the level of investment indirectly through taxes:

$$c_1^D < c_1^*;\ x^D < x^*$$

As one would expect, the reduction in repo market investment within a decentralized economy leads to less consumption in the good aggregate state for all repo market participants (both investors and borrowers):

$$c_{1\alpha}^D < c_{1\alpha}^*; c_{2\alpha}^D < c_{2\alpha}^*; b_{2\alpha}^D < b_{2\alpha}^*$$

The optimal repo rate is also affected through the reduction in investment:

$$r_\alpha^D > r_\alpha^*$$

With investment resources becoming more scarce, interests rates increase to clear the market.

The investment technology under discretion is under-utilized causing economy-wide consumption of the private good to be at an inefficiently low level compared to the Planner's case. However, in the bad aggregate state, higher taxes provide the fiscal authority with more resources for bailout funding – a mechanism that is less inefficient than prematurely selling collateral.

By Propositions 5 and 8, in the event that a fiscal crisis occurs, patient investors may have an incentive to withdraw early if $i \leq \theta$. To handle the excessive withdraws, financial intermediaries will begin liquidating collateral at rate p_F. In both the decentralized and social planner's cases, intermediaries' liquidity needs are accommodated with a bailout. The following comparison between cases highlights the benefits of higher taxation and the role of the government after a financial crisis is underway.

Liquidation of collateral is an inefficient means of raising resources. First, the liquidation price of collateral is less than both the repo price and the borrowers' value of collateral. In this way, use of the intermediaries' weak transformation technology can be thought of as a fire-sale. Second, premature sales of collateral adversely affect incentives to settle repos. For example, Proposition 1 shows that repos are re-priced in the bad aggregate state. An increase in repo rates during a crisis can distort incentives for borrowers to participate in the second leg of the repo. That is, fewer collateral goods upon repurchase encourages strategic default.

Bailout funding is based on the lost marginal utility from access to the economy's public good. On the other hand, during a fire-sale, the liquidation value of the collateral (p_F) may be low. Therefore, redistributing fiscal resources from the public good to the shadow banking system can prevent large economic losses:

$$c_{1\beta}^D > c_{1\beta}^*; c_{2\beta}^D > c_{2\beta}^*; b_{2\beta}^D > b_{2\beta}^*$$

As public liquidity is higher under discretion, $\psi_\tau^D > \psi_\tau^*$, the liquidity needs of intermediaries in a time of crisis are impacted by a greater degree under discretion than in a Planner-run financial system:

$$k^D > k^* \tag{13}$$

With more liquidity coming from bailouts, collateral liquidation is decreased:

$$\lambda_\beta^D < \lambda_\beta^* \tag{14}$$

Consequently, intermediaries are able to meet their withdraw obligations after a run without substantial liquidation of assets. Along with the increased stability of the repo market, the higher taxes in the decentralized economy also lessen the likelihood for runs to occur:

Proposition 11: $\Phi^* \supset \Phi^D$. *Furthermore, the decentralized financial system has a more stable repo market when crises occur.*

If a run by investors on MMMFs were to occur in either case, the decentralized system would be less likely to see a collapse of the repo market. Two characteristics of the decentralized economy create this featured stability. First, smaller initial investments by intermediaries in the repo market lead to lower returns. In this manner, policymakers face a trade-off between the efficiency of the shadow banking system versus its stability. Second, as $k^D > k^*$, the fiscal authority shoulders more of the burden than the Planner. Since the fiscal authority transfers more resources to intermediaries in the system, less collateral would be sold indicating the extent of "fire sales" would be lower. Consequently, repo rates will not be as volatile under discretion as in a Planner-run financial system. Therefore, the probability of a run occurring in the Planner's economy is strictly greater than in the decentralized case. That is, the Planner may be more willing to tolerate a crisis in the shadow banking system than fiscal authorities under discretion.

6 Conclusions

MMMFs are one of the primary ways both retail and institutional investors seek returns on their capital – a fact that has not escaped policymakers as the opening chapter of Dodd-Frank addresses the issue of increasing stability within non-bank financial institutions. Yet, failures of MMMFs are highly unlikely events. However, aggregate financial crises like that of 2007-2008 have shown to create runs across the mass of MMMFs which can lead to the failure of many funds at once. Without access to deposit insurance or the discount window, MMMFs must liquidate some of their illiquid assets in order to meet excessive early redemptions from runs.

In this paper we provide support for policies which can increase the stability of the repo market. It is important to note that these policy implications contrast with Title I of the Dodd-Frank Act which intends on eliminating the *expectation* of taxpayer funded bailouts. In particular, we find eliminating the expectation of bailouts increases the likelihood of runs and, in turn, can lead to a collapse of the repo market. Subsequently, extreme economic losses are borne by investors due to the extensive devaluation of the collateral assets held by MMMFs. Providing the expectation of bailouts, on the other hand, bolsters the repo market.

A benevolent planner choosing the efficient allocation of resources sets taxes lower than in a decentralized economy. This occurs because the fiscal authority only has one instrument to indirectly affect the amount of risk-sharing in the economy – the level of taxes. Lower taxes in the Planner's allocation generally lead to to higher returns for all repo market participants. By comparison, the higher level of taxes imposed by a fiscal authority with discretion leads to inefficiently low levels of investment and lower investor returns. Yet, in the event of a crisis, the fiscal authority has resources to capitalize institutions under distress.

In comparison to Keister (2014), public sector intervention in our model can play an important role in stabilizing the repo market by preventing massive liquidation of collateral which causes repo markets to collapse. Thus, policymakers aiming to increase stability of the repo market in times of crisis should carefully consider the use of public funds to stabilize shadow banking institutions. Nevertheless, as some liquidation of collateral is optimal, we also show that optimal policy does impose some losses among participants in the shadow banking system.

Understanding that bailouts for large financial institutions can be extremely unpopular, Proposition 10 gives policymakers an alternative. In particular, imposing minimum standards on haircuts (or haircut floors) have recently been proposed by the SEC Commissioner, Financial Stability Board, and European Parliment.[19] Proposition 10 offers support to these proposals. Higher minimum standards on haircuts lead to a lower probability of repo market collapse. Fur-

[19] Policy recommendations regarding haircut floors can be found in Financial Stability Board (2013) and Comotto (2013). Commissioner of the SEC, Kara M. Stein, recently encouraged the agency to require some meaningful haircuts on repos as well.

thermore, there is a level of haircuts in which a bailout would not be necessary in order to prevent a market collapse. Obviously, however, minimum standards would come at a cost. Higher haircuts place greater restrictions on the amount of funding borrowers can access through the repo market since borrowers can only pledge as much collateral as they have hold. Therefore, the extent of over-collateralization is limited by the amount of collateral resources available to borrowers.

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7 Appendix

Proof of Proposition 1. The planner's objective:

$$\max_{r_s}(1-\theta)(1-\widehat{\pi}_s)u\left(c_{2s}\right)+\eta\left(b_{2s}\right),$$

$$r_s=\left(\frac{(1-\theta)(1-\widehat{\pi}_s)}{1+(1-\theta)(1-\widehat{\pi}_s)}\right)\frac{R\cdot x+p_B\left(z-\lambda_s\delta x\right)}{(1-\lambda_s)x\delta}$$

Proof of Corollary 1:
The state-dependent levels of consumption for investors are:

$$c_{2s}(\lambda_s,x,\delta)=r_s\left(\frac{(1-\lambda_s)x\delta}{(1-\theta)(1-\widehat{\pi}_s)}\right)$$

$$c_{2s}(\lambda_s,x,\delta)=\frac{\left[R\cdot x+p_B\left(z-\lambda_s\delta x\right)\right]}{1+(1-\theta)(1-\widehat{\pi}_s)}$$

By comparison, for borrowers:

$$b_{2s}=R\cdot x-r_s(1-\lambda_s)x\delta+p_B\left(z-\lambda_s\delta x\right)$$

$$b_{2s}=R\cdot x-\left(\frac{(1-\theta)(1-\widehat{\pi}_s)}{1+(1-\theta)(1-\widehat{\pi}_s)}\right)\left[R\cdot x+p_B\left(z-\lambda_s\delta x\right)\right]+p_B\left(z-\lambda_s\delta x\right)$$

In the good state:

$$b_{2\alpha}=R\cdot x-\left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left[R\cdot x+p_Bz\right]+p_Bz$$

$$b_{2\alpha}=R\cdot x-(1-\theta)(1-\widehat{\pi}_\alpha)c_{2\alpha}+p_Bz$$

$$b_{2\alpha}=c_{2\alpha}$$

In the bad state:

$$b_{2\beta}=R\cdot x-(1-\theta)(1-\widehat{\pi}_\beta)c_{2\beta}+p_B\left(z-\lambda_\beta\delta x\right)$$

$$\frac{Rx+p_Bz-\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]c_{2\beta}}{p_B\delta x}=\lambda_\beta$$

$$b_{2\beta} = c_{2\beta}$$

Therefore, regardless of the state:

$$b_{2s} = c_{2s}$$

Proof of Proposition 2.
In the good aggregate state, the Planner's objective is:

$$\begin{aligned} V(\psi_\alpha; \widehat{\pi}_\alpha) \quad &\equiv \quad \max_{c_{1\alpha},\, \lambda_\alpha} (1-\theta)\left(\widehat{\pi}_\alpha u(c_{1\alpha}) + (1-\widehat{\pi}_\alpha) u\left(\frac{[R \cdot x + p_B(z - \lambda_\alpha \delta x)]}{1 + (1-\theta)(1-\widehat{\pi}_\alpha)}\right)\right) + \\ &\quad \mu_\alpha \left[p_F \lambda_\alpha \delta x + 1 - \tau - x - \theta c_1 - (1-\theta)\widehat{\pi}_\alpha c_{1\alpha}\right] \end{aligned}$$

It is optimal not to liquidate in the good state as long as:

$$\frac{dV(\psi_\alpha; \widehat{\pi}_\alpha)}{d\lambda_\alpha} = (1-\theta)(1-\widehat{\pi}_\alpha) u'(c_{2\alpha}) \left(\frac{-p_B \delta x}{1 + (1-\theta)(1-\widehat{\pi}_\alpha)}\right) + \mu_\alpha p_F \delta x < 0$$

That is, if the additional utility received by the impatient investors upon liquidation of the collateral is not sufficient to cover the loss among the patient investors, then none of the collateral will be liquidated.

By comparison, the optimal choice of $c_{1\alpha}$ *is pinned down by the shadow value of resources in state* α :

$$\frac{dV(\psi_\alpha; \widehat{\pi}_\alpha)}{dc_{1\alpha}} = u'(c_{1\alpha}) = \mu_\alpha$$

Liquidating the marginal unit of collateral would lower income that would be allocated to the late consumers, depending on the liquidation value that the borrower would be able to generate. The loss of consumption among the late consumers is given by $\frac{p_B \delta x}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}$. *The total loss of utility among the population of late consumers depends on the number remaining after the aggregate state has been revealed,* $(1-\theta)(1-\widehat{\pi}_\alpha)$.

$$\frac{dV(\psi_\alpha; \widehat{\pi}_\alpha)}{d\lambda_\alpha} = (1-\theta)(1-\widehat{\pi}_\alpha) u'(c_{2\alpha}) \left(\frac{-p_B}{1 + (1-\theta)(1-\widehat{\pi}_\alpha)}\right) + u'(c_{1\alpha}) p_F < 0$$

$$\frac{u'(c_{1\alpha})}{u'(c_{2\alpha})} < \left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1 + (1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left(\frac{p_B}{p_F}\right)$$

Proof of Corollary 2:

Defining the condition in terms of explicit consumption decisions:

$$\left(\frac{1-\tau-x-\theta c_1}{(1-\theta)\,\widehat{\pi}_\alpha}\right)^{-\gamma} < \left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left(\frac{p_B}{p_F}\right)\left(\frac{[R\cdot x+p_B z]}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)^{-\gamma}$$

Solving for x:

$$x < \frac{1-\tau-\theta c_1-p_B z\left(\frac{(1-\theta)\widehat{\pi}_\alpha}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\alpha)}{(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}}{\left[1+R\left(\frac{(1-\theta)\widehat{\pi}_\alpha}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\alpha)}{(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]}$$

Proof of Proposition 3:

The Planner's objective once the aggregate state has been revealed is equal to:

$$\begin{aligned} V\left(\psi_\beta;\widehat{\pi}_\beta\right) \equiv & \max_{c_{1\beta},\,c_{2\beta}} (1-\theta)\left(\widehat{\pi}_\beta u\left(c_{1\beta}\right)+\left(1-\widehat{\pi}_\beta\right)u\left(c_{2\beta}\right)\right)+ \\ & \mu_\beta\left[\frac{p_F}{p_B}Rx+p_F z+1-\tau+k-x-\theta c_1-\left\{(1-\theta)\widehat{\pi}_\beta c_{1\beta}+\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]c_{2\beta}\right\}\right] \end{aligned}$$

$$\frac{dV\left(\psi_\beta;\widehat{\pi}_\beta\right)}{dc_{1\beta}} = u'\left(c_{1\beta}\right)=\mu_\beta$$

$$\frac{dV\left(\psi_\beta;\widehat{\pi}_\beta\right)}{dc_{2\beta}} = u'\left(c_{2\beta}\right)=\frac{p_F}{p_B}\frac{\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]}{(1-\theta)\left(1-\widehat{\pi}_\beta\right)}\mu_\beta$$

Therefore,

$$\frac{u'\left(c_{1\beta}\right)}{u'\left(c_{2\beta}\right)}=\frac{p_B}{p_F}\frac{(1-\theta)\left(1-\widehat{\pi}_\beta\right)}{\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]}$$

Proof of Corollary 3

First, we solve for the condition that $\lambda_\beta < 1$. We do so by taking the solutions for $c_{1\beta}$ and $c_{2\beta}$ and maximizing over λ_β :

$$V\left(\psi_\beta;\widehat{\pi}_\beta\right)\equiv\max_{\lambda_\beta}\left(1-\theta\right)\left[\widehat{\pi}_\beta u\left(\frac{p_F\lambda_\beta\delta x+1-\tau+k-x-\theta c_1}{(1-\theta)\widehat{\pi}_\beta}\right)+\left(1-\widehat{\pi}_\beta\right)u\left(\frac{\left[R\cdot x+p_B\left(z-\lambda_\beta\delta x\right)\right]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\right]$$

Evaluating at $\lambda_\beta=1$ and establishing conditions in which 100% liquidation is not optimal:

$$\begin{aligned}&u'\left(\frac{p_F\delta x+1-\tau+k-x-\theta c_1}{(1-\theta)\widehat{\pi}_\beta}\right)p_F\delta x\\ <\quad&\left(1-\theta\right)\left(1-\widehat{\pi}_\beta\right)u'\left(\frac{\left[R\cdot x+p_B\left(z-\delta x\right)\right]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_B\delta x}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\end{aligned}$$

$$\left(\frac{p_F}{p_B}\right)\left(\frac{\left[R\cdot x+p_B\left(z-\delta x\right)\right]}{p_F\delta x+1-\tau+k-x-\theta c_1}\right)^\gamma<\frac{(1-\widehat{\pi}_\beta)}{(\widehat{\pi}_\beta)^\gamma}\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)}\right)^{\gamma-1}$$

Second, we solve for the condition that $\lambda_\beta>0$ by evaluating at $\lambda_\beta=0$:

$$\begin{aligned}&\left(1-\theta\right)\widehat{\pi}_\beta u'\left(\frac{1-\tau+k-x-\theta c_1}{(1-\theta)\widehat{\pi}_\beta}\right)\left(\frac{p_F\delta x}{(1-\theta)\widehat{\pi}_\beta}\right)\\ >\quad&\left(1-\theta\right)\left(1-\widehat{\pi}_\beta\right)u'\left(\frac{\left[R\cdot x+p_B z\right]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_B\delta x}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\end{aligned}$$

$$\left(\frac{p_F}{p_B}\right)\left(\frac{\left[R\cdot x+p_B z\right]}{1-\tau+k-x-\theta c_1}\right)^\gamma>\frac{(1-\widehat{\pi}_\beta)}{(\widehat{\pi}_\beta)^\gamma}\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)}\right)^{\gamma-1}$$

With the two conditions above, we can reduce the inequalities down to thresholds on the exogenous parameter p_F.

$$p_F<\overline{p_F}=p_B\frac{(1-\widehat{\pi}_\beta)}{(\widehat{\pi}_\beta)^\gamma}\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)}\right)^{\gamma-1}\left(\frac{p_F\delta x+1-\tau+k-x-\theta c_1}{\left[R\cdot x+p_B\left(z-\delta x\right)\right]}\right)^\gamma$$

$$p_F>\underline{p_F}=p_B\frac{(1-\widehat{\pi}_\beta)}{(\widehat{\pi}_\beta)^\gamma}\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)}\right)^{\gamma-1}\left(\frac{1-\tau+k-x-\theta c_1}{\left[R\cdot x+p_B z\right]}\right)^\gamma$$

Proof of Corollary 4:

The lower MRS can essentially be described by the fraction of patient investors in the economy after the bad state occurs as opposed to the good state.

$$\frac{(1-\theta)(1-\widehat{\pi}_{\beta})}{[1+(1-\theta)(1-\widehat{\pi}_{\beta})]} < \frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{[1+(1-\theta)(1-\widehat{\pi}_{\alpha})]}$$

After substituting in the values for $\widehat{\pi}_{\alpha}$ and $\widehat{\pi}_{\beta}$, we get

$$(1-\theta)+(1-\theta)(1-\pi) < 1+(1-\theta)(1-\pi)$$

Derivation of the Condition in Corollary 5.

We know that $u'(c_{1\alpha}) = \mu_{\alpha}$ and $u'(c_{2\alpha})\left[\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\right] > \frac{p_F}{p_B}\mu_{\alpha}$

$$u'(c_{1\alpha}) = u'(c_{2\alpha})\,R\left[\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\right]$$

$$\mu_{\alpha} > R\frac{p_F}{p_B}\mu_{\alpha}$$

$$\frac{p_F}{p_B}R < 1$$

Proof of Planner's Tax:

$$\underset{\tau}{Max}\ \theta u(c_1) + [V(1-\tau-x-\theta c_1;\widehat{\pi}_{\alpha}) + v(g_{\alpha})]$$

The optimal tax in the good aggregate state follows:

$$\underset{\tau}{Max}\ \ [V(1-\tau-x-\theta c_1;\widehat{\pi}_{\alpha}) + v(g_{\alpha})]$$

$$\frac{dV(1-\tau-x-\theta c_1;\widehat{\pi}_{\alpha})}{d\tau} + v'(g_{\alpha}) = 0$$

Recall

$$\begin{aligned} V(\psi_{\alpha};\widehat{\pi}_{\alpha}) \quad \equiv \quad & (1-\theta)(\widehat{\pi}_{\alpha}u(c_{1\alpha}) + (1-\widehat{\pi}_{\alpha})\,u(c_{2\alpha})) + \\ & \mu_{\alpha}\left[\frac{p_F}{p_B}Rx + p_F z + 1 - \tau - x - \theta c_1 - \left[(1-\theta)\widehat{\pi}_{\alpha})c_{1\alpha} + \frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_{\alpha})\right]c_{2\alpha}\right]\right] \end{aligned}$$

$$\frac{dV(\psi_{\alpha};\widehat{\pi}_{\alpha})}{d\tau} = -\mu_{\alpha}$$

Plugging these values in:

$$\mu_{\alpha} = v'(\tau)$$

Proof of the Planner's Solutions

To move further through our analysis, we need to find solutions for our variables of interest. We begin with the consumption allocations:

$$c_{1\beta}(\lambda_\beta, x, \delta, g_\beta) = \frac{p_F \lambda_\beta \delta x + 1 - \tau + k - x - \theta c_1}{(1-\theta)\widehat{\pi}_\beta}$$

$$c_{2\beta}(\lambda_\beta, x, \delta) = \frac{[R \cdot x + p_B\,(z - \lambda_\beta \delta x)]}{1 + (1-\theta)(1-\widehat{\pi}_\beta)}$$

Now, solving the the collateral liquidation condition:

$$u'\left(c_{1\beta}(\lambda_\beta, x, \delta)\right) = \left(\frac{(1-\theta)(1-\widehat{\pi}_\beta)}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_B}{p_F}\right) u'\left(c_{2\beta}(\lambda_\beta, x, \delta)\right)$$

$$c_{1\beta} = \left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}} c_{2\beta}$$

$$\frac{p_F \lambda_\beta \delta x + 1 - \tau + k - x - \theta c_1}{(1-\theta)\widehat{\pi}_\beta} = \left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}} \frac{[R \cdot x + p_B\,(z - \lambda_\beta \delta x)]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}$$

$$\lambda_\beta = \frac{\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\left[\frac{[R\cdot x + p_B z]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right] - \frac{1-\tau+k-x-\theta c_1}{(1-\theta)\widehat{\pi}_\beta}}{\left[\frac{p_F \delta x}{(1-\theta)\widehat{\pi}_\beta} + \left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}} \frac{p_B \delta x}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right]}$$

Using the feasibility constraint constructed for the bad state, we can solve for consumption across investors:

$$\frac{p_F}{p_B} Rx + p_F z + 1 - g_\beta - x - \theta c_1 = \left[(1-\theta)\widehat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right] c_{2\beta}\right]$$

From the first-order condition,

$$c_{1\beta} = \left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}} c_{2\beta},$$

we obtain

$$\frac{p_F}{p_B} Rx + p_F z + 1 - g_\beta - x - \theta c_1 = \left[(1-\theta)\widehat{\pi}_\beta + \frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{-\frac{1}{\gamma}}\right] c_{1\beta}$$

Applying the relationship for the public good,

$$u'\left(c_{1\beta}\right) = \mu_{\beta} = u'(g_{\beta})$$

$$c_{1\beta} = g_{\beta}$$

and solving for $c_{1\beta}$*, we obtain:*

$$c_{1\beta} = \frac{\frac{p_F}{p_B}Rx + p_F z + 1 - x - \theta c_1}{\left[1 + (1-\theta)\widehat{\pi}_{\beta} + \frac{p_F}{p_B}\left[1 + (1-\theta)(1-\widehat{\pi}_{\beta})\right]\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_{\beta})}{(1-\theta)(1-\widehat{\pi}_{\beta})}\right)\left(\frac{p_F}{p_B}\right)\right]^{-\frac{1}{\gamma}}\right]}$$

Then, using the first-order conditon once again we are able to find $c_{2\beta}$*.*

$$c_{2\beta} = c_{1\beta}\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_{\beta})}{(1-\theta)(1-\widehat{\pi}_{\beta})}\right)\left(\frac{p_F}{p_B}\right)\right]^{-\frac{1}{\gamma}}$$

Consumption solutions in the good state are given by:

$$c_{1\alpha}(c_1, x, g_{\alpha}) = \frac{1 - g_{\alpha} - x - \theta c_1}{(1-\theta)\,\widehat{\pi}_{\alpha}}$$

$$c_{2\alpha} = \frac{\left[R \cdot x + p_B z\right]}{1 + (1-\theta)\left(1 - \widehat{\pi}_{\alpha}\right)}$$

Since we know that

$$u'\left(c_{1\alpha}\right) = \mu_{\alpha}$$

$$v'(g_{\alpha}) = \mu_{\alpha}$$

$$u'\left(c_1\right) = \mu_{\alpha}$$

Therefore,

$$c_{1\alpha}(x, c_1) = \frac{1-x}{\left(1 + \theta + (1-\theta)\,\widehat{\pi}_{\alpha}\right)}$$

and

$$c_1 = \frac{1-x}{\left(1 + \theta + (1-\theta)\,\widehat{\pi}_{\alpha}\right)}$$

The investment condition yields

$$u'\left(c_{2a}\right)R\left[\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1 + (1-\theta)\left(1-\widehat{\pi}_{\alpha}\right)}\right] = u'\left(c_{1\alpha}\right)$$

$$c_{2a}\left[R\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)\,(1-\widehat{\pi}_{\alpha})}\right]^{-\frac{1}{\gamma}}=c_{1\alpha}$$

Recall

$$c_{1\alpha}=c_{1}=\frac{1-x}{1+\theta+(1-\theta)\,\widehat{\pi}_{\alpha}},$$

By simply substituting into the investment condition the solutions for c_1 *and* $c_{2\alpha}$*, the solution for* x *is shown to be:*

$$x=\frac{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_{\alpha}}-\frac{p_{B}z}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\left[R\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\right]^{-\frac{1}{\gamma}}}{\left(\frac{R}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\left[R\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\right]^{-\frac{1}{\gamma}}+\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_{\alpha}}\right)}$$

Proof of Proposition 4.

In order for settlement to occur in the second leg of the repo, the borrower's incentive constraint must be satified:

$$r_{s}<p_{B}.$$

After substituting the solution for the repo rate in state s*:*

$$r_{s}=\left(\frac{(1-\theta)(1-\widehat{\pi}_{s})}{1+(1-\theta)(1-\widehat{\pi}_{s})}\right)\frac{R\cdot x+p_{B}z-p_{B}\lambda_{s}x\delta}{(1-\lambda_{s})x\delta}$$

and solving for λ_s *we obtain*

$$\lambda_{s}<1-(1-\theta)(1-\widehat{\pi}_{s})\left[\frac{R\cdot x+p_{B}z}{x\delta p_{B}}-1\right].$$

Consequently, the condition for settlement can be written in terms of an upper-bound on the liquidation of collateral.

We begin by verifying the condition is satisfied in the good state where $\lambda_{\alpha}=0:$

$$0<\left[1+(1-\theta)(1-\widehat{\pi}_{\alpha})\right]x\delta p_{B}-(1-\theta)(1-\widehat{\pi}_{\alpha})\left(R\cdot x+p_{B}z\right)$$

$$\left[\delta p_{B}-\left(\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\right)R\right]x^{*}>\left(\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)(1-\widehat{\pi}_{\alpha})}\right)p_{B}z$$

Given that

$$x^* = \frac{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} - \frac{p_B z}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\left[R\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right]^{-\frac{1}{\gamma}}}{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} + \frac{R}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\left[R\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right]^{-\frac{1}{\gamma}}}$$

we find

$$\begin{aligned}\delta \quad > \quad & \underline{\delta_\alpha^*} \equiv \frac{R}{p_B}\left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right) \\ & +z\cdot\left[\frac{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} + \frac{R}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\left[R\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right]^{-\frac{1}{\gamma}}}{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} - \frac{p_B z}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\left[R\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right]^{-\frac{1}{\gamma}}}\right]\left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)\end{aligned}$$

Since x^* is not a function of δ and is entirely in terms of exogenous parameters, the threshold can be simplified:

$$\delta > \left(\frac{R}{p_B} + \frac{z}{x^*}\right)\left(\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right)$$

Next, for the bad state. *Recall that the solution for λ_β is:*

$$\lambda_\beta = \frac{\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\left[\frac{[R\cdot x+p_B z]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right] - \frac{1-\tau+k-x-\theta c_1}{(1-\theta)\widehat{\pi}_\beta}}{\left[\frac{p_F\delta x}{(1-\theta)\widehat{\pi}_\beta} + \left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\frac{p_B\delta x}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right]}$$

Plugging in the solution for λ_β into

$$\lambda_s < 1-(1-\theta)(1-\widehat{\pi}_s)\left[\frac{R\cdot x+p_B z}{x\delta p_B} - 1\right],$$

we find

$$\begin{aligned}\tau \quad < \quad & (1-x-\theta c_1)+k+\left(\frac{p_F}{p_B}\right)x\delta p_B \\ & -\left[\left(\frac{p_F}{p_B}\right)(1-\theta)(1-\widehat{\pi}_\beta)+(1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right][R\cdot x+p_B(z-x\delta)]\end{aligned}$$

From the first-order conditions in the planner's case, we know that tax imposed in period 0 is

$$\tau = c_1.$$

In addition, from the previous solutions:

$$c_1 = \frac{1-x}{(1+\theta+(1-\theta)\,\widehat{\pi}_\alpha)}.$$

Plugging these equations into the following inequality

$$\begin{aligned}(1+\theta)\,c_1 \quad < \quad & (1-x)+k+\left(\frac{p_F}{p_B}\right)x\delta p_B \\ & -\left[\left(\frac{p_F}{p_B}\right)(1-\theta)(1-\widehat{\pi}_\beta)+(1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right][R\cdot x+p_B\,(z-x\delta)]\end{aligned}$$

yields:

$$\begin{aligned}\delta \quad > \quad \underline{\delta_\beta^*} = & \frac{\left(R+\frac{p_B z}{x^*}\right)\left[\left(\frac{p_F}{p_B}\right)(1-\theta)(1-\widehat{\pi}_\beta)+(1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]}{\left[p_F\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]+(1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]} \\ & + \frac{\left(1-\frac{1}{x^*}\right)\frac{(1-\theta)\widehat{\pi}_\alpha}{(1+\theta+(1-\theta)\widehat{\pi}_\alpha)}-\frac{k^*}{x^*}}{\left[p_F\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]+(1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]}\end{aligned}$$

Therefore, in order for settlement to take place in either aggregate state, $\delta > \max(\underline{\delta_\alpha^*}, \underline{\delta_\beta^*})$.

Proof of Proposition 5:

The proof of the Proposition relates to showing the following relationship hold true

$$\frac{\widetilde{c}_1}{\widetilde{c}_{2\beta}} \leq \frac{c_1^*}{c_{2\beta}^*}$$

and is broken down into three steps.

Step 1. The Planner always chooses $x = x^*$ *and* $c_1 = c_1^*$.

From the first-order conditions in period 0 and 1, the relationship between impatient consumption is found to be

$$c_{1\alpha} = c_1.$$

Therefore, utilizing this relationship along with the investment condition:

$$u'\left(c_{1\alpha}\right) = u'\left(c_{2\alpha}\right)R\left[\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)\left(1-\widehat{\pi}_\alpha\right)}\right],$$

By CRRA preferences it must be true that

$$c_1 < c_{2\alpha}$$

so long as $R\left[\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)}\right] > 1$.

Thus, a patient investor with $i \leq \theta$ *will strictly prefer to wait in state* α. *Since the realization of the bad state is unexpected, decisions ex-ante are made only with regards to the strategy that investors choose in the good state. As no patient investor has an incentive to run in the good state then it directly follows that the social planner will always construct* $x = x^*$ *and* $c_1 = c_1^*$.

Step 2. Show that $\widetilde{\mu_\beta} < \mu_\beta^*$.

Due to CRRA preferences, we may write:

$$\phi c_{1\beta} = c_{2\beta}$$

for some scalar ϕ.

From the feasibility constraint, the proportion of the population that is left to be funded after the aggregate state is revealed is given by:

$$\left[((1-\theta)\widehat{\pi}_\beta)c_{1\beta} + \frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]c_{2\beta}\right]$$

Using the first-order condition

$$c_{1\beta} = \left[\frac{p_F}{p_B}\frac{\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]}{(1-\theta)(1-\widehat{\pi}_\beta)}\right]^{-\frac{1}{\gamma}} c_{2\beta}$$

Let $\left[\frac{p_F}{p_B}\frac{[1+(1-\theta)(1-\widehat{\pi}_\beta)]}{(1-\theta)(1-\widehat{\pi}_\beta)}\right]^{-\frac{1}{\gamma}} = \phi$. *Then* $\left[(1-\theta)\widehat{\pi}_\beta + \frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\phi\right]c_{1\beta}$ *is increasing increasing in* $\widehat{\pi}_\beta$ *for any level of risk aversion.*

The feasibility constraint for the bad state is

$$p_F z + 1 - \left(1-\frac{p_F}{p_B}R\right)x - g_\beta - \theta c_1 - \left[(1-\theta)\widehat{\pi}_\beta + \phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\right]c_{1\beta} = 0,$$

Since the Planner chooses c_1^* *and* x^* *and, in turn the investors always follow the strategy outlined by* y^*, *the feasibility constraint across strategy profiles in the bad state can be reduced to:*

$$\widetilde{g_\beta} + \left[(1-\theta)\widetilde{\pi}_\beta + \phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widetilde{\pi}_\beta)\right]\right]\widetilde{c_{1\beta}} = g_\beta^* + \left[(1-\theta)\widehat{\pi}_\beta + \phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\right]c_{1\beta}^*.$$

As $\left[(1-\theta)\widetilde{\pi}_\beta) + \phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widetilde{\pi}_\beta)\right]\right] < \left[(1-\theta)\widehat{\pi}_\beta + \phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\right]$, *then it follows that one of the following must hold:*

$$\widetilde{c_{1\beta}} > c^*_{1\beta}; \widetilde{g_\beta} > g^*_\beta$$

Both of which imply that $\widetilde{\mu_\beta} < \mu^*_\beta$.

Step 3.

By Steps 1 and 2,

$$\frac{u'(c^*_1)}{\widetilde{\mu}_\beta} > \frac{u'(c^*_1)}{\mu^*_\beta}.$$

Substituting in the optimal risk-sharing condition for the bad state:

$$\mu^*_\beta = \left(\frac{(1-\theta)\,(1-\widehat{\pi}_\beta)}{1+(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_B}{p_F}\right)u'(c^*_{2\beta})$$

$$\widetilde{\mu_\beta} = \left(\frac{(1-\theta)\,(1-\widetilde{\pi}_\beta)}{1+(1-\theta)(1-\widetilde{\pi}_\beta)}\right)\left(\frac{p_B}{p_F}\right)u'(\widetilde{c_{2\beta}})$$

We find

$$\left(\frac{u'(c^*_1)}{u'(\widetilde{c_{2\beta}})}\right) \geq \left(\frac{u'(c^*_1)}{u'(c^*_{2\beta})}\right)$$

Therefore, $\frac{\widetilde{c_1}}{\widetilde{c_{2\beta}}} \leq \frac{c^*_1}{c^*_{2\beta}}$. *This completes the proof of fragility.*

Proof of Proposition 6:

Using the relationship between consumption among the impatient investors once again,

$$c_1 = c_{1\alpha}$$

Then, combine the result from the run condition:

$$c_{1\alpha} > c_{2\beta}.$$

Imposing CRRA preferences and the period 1 first-order conditions, the shadow values for each state are:

$$\mu^*_\alpha < \mu^*_\beta.$$

Thus, public good provision across states must be defined by

$$\tau^* - k < \tau^*.$$

The condition for collateral liquidation is simply the lower bound found in Corollary 3.

Proofs for the Decentralized Economy

The Impact of Taxes on Investment:
From the Investment Condition:

$$u''\left(c_{2\alpha}^{D}\right)R\left[\frac{(1-\theta)(1-\widehat{\pi}_{\alpha})}{1+(1-\theta)\left(1-\widehat{\pi}_{\alpha}\right)}\right]\frac{dc_{2\alpha}^{D}}{d\tau^{D}}=u''\left(c_{1\alpha}\right)\frac{dc_{1\alpha}^{D}}{d\tau^{D}}$$

From the Period 2 Budget Constraint:

$$\left[1+(1-\theta)\left(1-\widehat{\pi}_{\alpha}\right)\right]\frac{dc_{2\alpha}^{D}}{d\tau^{D}}=R\frac{dx^{D}}{d\tau^{D}}$$

From the Period 1 Budget Constraint:

$$(1-\theta)\,\widehat{\pi}_{\alpha}\frac{dc_{1\alpha}^{D}}{d\tau^{D}}=-1-\frac{dx^{D}}{d\tau^{D}}-\theta\frac{dc_{1}^{D}}{d\tau^{D}}$$

From the Period 0 First-Order Condition:

$$u''\left(c_{1\alpha}^{D}\right)\frac{dc_{1\alpha}^{D}}{d\tau^{D}}=u''\left(c_{1}^{D}\right)\frac{dc_{1}^{D}}{d\tau^{D}}$$

In order to isolate the impact of taxes on investment, we combine all of the aforementioned conditons

$$(1-\theta)\,\widehat{\pi}_{\alpha}\frac{dc_{1\alpha}^{D}}{d\tau^{D}}=-1-\frac{dx^{D}}{d\tau^{D}}-\theta\frac{u''\left(c_{1\alpha}^{D}\right)}{u''\left(c_{1}^{D}\right)}\frac{dc_{1\alpha}^{D}}{d\tau^{D}}$$

and solve for $dc_{1\alpha}^{D}/d\tau^{D}$.

$$\frac{dx^{D}}{d\tau^{D}}=\frac{-1}{\left[1+(1-\theta)\,\widehat{\pi}_{\alpha}\frac{u''\left(c_{2\alpha}^{D}\right)}{u''\left(c_{1\alpha}^{D}\right)}\frac{R^{2}(1-\theta)(1-\widehat{\pi}_{\alpha})}{[1+(1-\theta)(1-\widehat{\pi}_{\alpha})]^{2}}+\theta\frac{u''\left(c_{2\alpha}^{D}\right)}{u''\left(c_{1\alpha}^{D}\right)}\frac{R^{2}(1-\theta)(1-\widehat{\pi}_{\alpha})}{[1+(1-\theta)(1-\widehat{\pi}_{\alpha})]^{2}}\right]}$$

Let $\chi=\frac{u''\left(c_{2\alpha}^{D}\right)}{u''\left(c_{1\alpha}^{D}\right)}\frac{R^{2}(1-\theta)(1-\widehat{\pi}_{\alpha})}{[1+(1-\theta)(1-\widehat{\pi}_{\alpha})]^{2}}$ *so that*

$$\frac{dx^{D}}{d\tau^{D}}=\frac{-1}{\left[1+\left((1-\theta)\,\widehat{\pi}_{\alpha}+\theta\right)\chi\right]}$$

$$-1<\frac{dx^{D}}{d\tau^{D}}<0.$$

Proof of Portfolio Choice between the Planner and Decentralized Cases

Suppose that $c_1^D \geq c_1^*$. *From the feasibility constraint:*

$$(1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

$$(1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha} + x = 1 - \tau - \theta c_1$$

If $c_1^D > c_1^*$ *and we know* $\tau^D > \tau^*$ *then either* $x^D < x^*$ *or* $c_{1\alpha}^D < c_{1\alpha}^*$. *Both of which lead to* $\mu_\alpha^D > \mu_\alpha^*$ *which contradicts* $c_1^D \geq c_1^*$. *Therefore, it must be true that* $c_1^D < c_1^*$.

In turn from the investment condition:

$$\mu_\alpha = u'\left(c_{2\alpha}\right) R \left[\frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)\left(1-\widehat{\pi}_\alpha\right)}\right]$$

which implies that since $c_1^D < c_1^*$ *then* $x^D < x^*$.

Proof of a Higher Tax Rate

Recall:

$$u'(\tau^*) = \mu_\alpha^*$$

$$u'(\tau^D) = \mu_\alpha^D \left[1 + \frac{dx^D}{d\tau^D}\right]$$

Suppose that $\tau^D < \tau^*$. *Using the equations above, this means that*

$$\mu_\alpha^D \left[1 + \frac{dx^D}{d\tau^D}\right] > \mu_\alpha^*$$

or alternatively, since we know $-1 < \frac{dx^D}{d\tau} < 0$

$$\mu_\alpha^D > \mu_\alpha^*$$

This inequality also implies that $c_1^D < c_1^*$ *and* $x^D < x^*$. *From the feasibility constraint*

$$(1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha}^D + \tau^D = 1 - x^D - \theta c_1^D$$

Therefore, we must have $c_{1\alpha}^D > c_{1\alpha}^*$ *or* $\tau^D > \tau^*$. *Both of which are a contradiction. Consequently:* $\tau^D > \tau^*$.

Proof of Public Liquidity

Since $-1 < \frac{dx^D}{d\tau^D} < 0$, *then*

$$\frac{\mu_\alpha^*}{\mu_\alpha^*} > \frac{\mu_\alpha^D}{\mu_\alpha^D} + \frac{dx}{d\tau}$$

From the fiscal authority's tax decision,

$$u'(\tau^D) = \mu_\alpha^D \left[1 + \frac{dx}{d\tau}\right]$$

Therefore

$$\frac{u'(\tau^*)}{\mu_\alpha^*} > \frac{u'(\tau^D)}{\mu_\alpha^D}.$$

or

$$\frac{u'(\tau^*)}{u'\left(c_{1\alpha}^*\right)} > \frac{u'(\tau^D)}{u'\left(c_{1\alpha}^D\right)}.$$

Due to CRRA preferences

$$\frac{\tau^*}{c_{1\alpha}^*} < \frac{\tau^D}{c_{1\alpha}^D}$$

along with the budget constraint for period 1 in the good state:

$$(1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

$$(1-\theta)\,\widehat{\pi}_\alpha \frac{c_{1\alpha}}{\tau} + 1 = \frac{1 - x - \theta c_1}{\tau} = \psi_\tau^{-1}$$

Using the inequality $\frac{\tau^*}{c_{1\alpha}^*} < \frac{\tau^D}{c_{1\alpha}^D}$ *implies that* $\psi_\tau^* < \psi_\tau^D$.

Proof of Proposition 7
The proof of this Proposition follows directly from that of Proposition 4.

Proof of Proposition 8
The proof of this Proposition follows drectly from that of Proposition 5.

Bailouts vs. No Bailouts

Proof that $c_{2\beta}^{NB} < c_{2\beta}^*$.

The conditions that $c_1^{NB} < c_1^*$ *and* $x^{NB} < x^*$ *follow (except for notation) from the decentralized case. Using the feasibility constraints for the bad state:*

$$p_F z + 1 - \left[1 - \frac{p_F}{p_B} R\right] x^* - \theta c_1^* = \left[(1-\theta)\widehat{\pi}_\beta) + \phi \frac{p_F}{p_B}\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right]\right] c_{1\beta}^* + \tau^* - k^*$$

$$p_F z + 1 - \left[1 - \frac{p_F}{p_B} R\right] x^{NB} - \theta c_1^{NB} = \left[(1-\theta)\widehat{\pi}_\beta + \phi \frac{p_F}{p_B}\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right]\right] c_{1\beta}^{NB} + \tau^{NB}$$

We know that $\tau^{NB} = \tau^{D} > \tau^{*} > (\tau^{*} - k^{*})$. *Since* $c_1^{NB} < c_1^{*}$ *and* $x^{NB} < x^{*}$, *the relationship between* $c_{1\beta}^{NB}$ *and* $c_{1\beta}^{*}$ *is not explicitly defined here.*

Suppose that $c_{1\beta}^{NB} > c_{1\beta}^{*}$. *This also directly implies that* $c_{2\beta}^{NB} > c_{2\beta}^{*}$.

From the solution for $c_{2\beta}$:

$$c_{2\beta} = \frac{[R \cdot x + p_B (z - \lambda_\beta \delta x)]}{1 + (1-\theta)(1-\widehat{\pi}_\beta)}$$

Therefore, the only way that $c_{2\beta}^{NB} > c_{2\beta}^{*}$ *when* $x^{NB} < x^{*}$ *is if* $\lambda_\beta^{NB} < \lambda_\beta^{*}$.

Recall the collateral liquidation variable is defined by

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

The provision of public goods in the bad state is redefined as $\tau - k$; *where* k *once again is the size of the bailout package.*

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau - x - \theta c_1) - k}{p_F \delta x} = \lambda_\beta$$

The inequality above would then imply that

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta}^{NB} - \left(1 - \tau^{NB} - x^{NB} - \theta c_1^{NB}\right) - k^{NB}}{p_F \delta x^{NB}} < \frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta}^{*} - (1 - \tau^{*} - x^{*} - \theta c_1^{*}) - k^{*}}{p_F \delta x^{*}}$$

Since $k^{NB} = 0$,

$$(1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

Plugging this in

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta}^{NB} - (1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha}^{NB}}{p_F \delta x^{NB}} < \frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta}^{*} - (1-\theta)\,\widehat{\pi}_\alpha c_{1\alpha}^{*} - k^{*}}{p_F \delta x^{*}}$$

From the inequalities in the the no bailouts economy we have: $c_{1\beta}^{NB} > c_{1\beta}^{*}$, $c_{1\alpha}^{NB} < c_{1\alpha}^{*}$, *and* $x^{NB} < x^{*}$.

Therefore, the inequality cannot hold and thus it must be true that $c_{1\beta}^{NB} < c_{1\beta}^{*}$, $c_{2\beta}^{NB} < c_{2\beta}^{*}$, $\lambda_\beta^{NB} > \lambda_\beta^{*}$, and $\lambda_\beta^{NB} > \lambda_\beta^{D}$.

Proof of Proposition 9

Patient investor consumption is defined by

$$c_{2\beta} = \frac{[R \cdot x + p_B\,(z - \lambda_\beta \delta x)]}{1 + (1-\theta)(1-\widehat{\pi}_\beta)}$$

Since $\lambda_\beta^{NB} > \lambda_\beta^D$, $c_{2\beta}^{NB} < c_{2\beta}^D$. *With the level of* c_1 *the same regardless of bailouts, then*

$$c_1^{NB} = c_1^D$$

$$c_1^D \geq c_{2\beta}^D$$

implies that

$$c_1^{NB} > c_{2\beta}^{NB}$$

Since the inequality is strict, this implies there exists economies in Φ^{NB} *that would not be a part of* Φ^D. *Therefore,* Φ^{NB} *is a superset of* Φ^D.

The no bailouts regime contains all of the downfalls of the bailouts regime as well as added fragility in the event of the bad state. In the no bailouts regime, all of the consumption variables for private good consumption are chosen the same as those in the bailouts regime; however, when the bad state occurs consumption for patient investors is lower – leading to a broader set of run parameters.

Proof of Proposition 10

This is the stability condition for the decentralized case. The threshold for stability follows nearly exactly to that shown in Proposition 4.

$$\begin{aligned}\delta \quad > \quad \underline{\delta_\beta^D} = & \frac{\left(R + \frac{p_B z}{x^D}\right)\left[\left(\frac{p_F}{p_B}\right)(1-\theta)(1-\widehat{\pi}_\beta) + (1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]}{\left[p_F\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right] + (1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]} \\ & + \frac{\left(1 - \frac{1}{x^D}\right)\frac{(1-\theta)\widehat{\pi}_\alpha}{\left(\left[1 + \frac{dx^D}{d\tau^D}\right]^{-\frac{1}{\gamma}} + \theta + (1-\theta)\widehat{\pi}_\alpha\right)} - \frac{k^D}{x^D}}{\left[p_F\left[1 + (1-\theta)(1-\widehat{\pi}_\beta)\right] + (1-\theta)\widehat{\pi}_\beta\left[\left(\frac{1+(1-\theta)(1-\widehat{\pi}_\beta)}{(1-\theta)(1-\widehat{\pi}_\beta)}\right)\left(\frac{p_F}{p_B}\right)\right]^{\frac{1}{\gamma}}\right]}\end{aligned}$$

Since we know the only difference between bailouts and no bailouts is $k^{NB} = 0$, *then it must be true that for all* k^D *non-negligible from zero there exists a* δ *where* $\underline{\delta_\beta^D} < \delta < \underline{\delta_\beta^{NB}}$.

Comparing between the Decentralized and Planner's Allocations:

Using the feasibility constraints for the bad state

$$p_F z+1-\left[1-\frac{p_F}{p_B}R\right]x^*-\theta c_1^* = \left[(1-\theta)\widehat{\pi}_\beta)+\phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\right]c_{1\beta}^*+\tau^*-k^*$$

$$p_F z+1-\left[1-\frac{p_F}{p_B}R\right]x^D-\theta c_1^D = \left[(1-\theta)\widehat{\pi}_\beta+\phi\frac{p_F}{p_B}\left[1+(1-\theta)(1-\widehat{\pi}_\beta)\right]\right]c_{1\beta}^D+\tau^D-k^D$$

Since $c_1^D < c_1^*$ *and* $x^D < x^*$, *then in order for the above conditions to hold with equality* $c_{1\beta}^D > c_{1\beta}^*$ *or* $\left(\tau^D - k^D\right) > (\tau^* - k^*)$. *Both of which imply that* $\mu_\beta^D < \mu_\beta^*$. *This also directly implies that* $c_{2\beta}^D > c_{2\beta}^*$.

From the equation for $c_{2\beta}$ *we know*

$$c_{2\beta} = \frac{\left[R\cdot x + p_B\left(z-\lambda_\beta\delta x\right)\right]}{1+(1-\theta)(1-\widehat{\pi}_\beta)}$$

Therefore, the only way $c_{2\beta}^D > c_{2\beta}^*$ *when* $x^D < x^*$ *is if* $\lambda_\beta^D < \lambda_\beta^*$.

The economy in the decentralized state can be defined by: $\mu_\alpha^D > \mu_\alpha^*$, $\mu_\beta^D < \mu_\beta^*$, $c_1^D < c_1^*$, $x^D < x^*$, $\tau^D > \tau^*$, $\left(\tau^D - k^D\right) > (\tau^* - k^*)$, $c_{1\alpha}^D < c_{1\alpha}^*$, $c_{2\alpha}^D < c_{2\alpha}^*$, $c_{1\beta}^D > c_{1\beta}^*$, $c_{2\beta}^D > c_{2\beta}^*$, and $\lambda_\beta^D < \lambda_\beta^*$. *In addition, using* (3) *the repo rate with respect to investment is shown to be:*

$$\frac{\partial r_\alpha}{\partial x} = -\frac{p_B z}{\delta x^2}$$

Proof that Bailouts are larger in the decentralized case
The collateral liquidation variable is defined by:

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1-\tau+k-x-\theta c_1)}{p_F\delta x} = \lambda_\beta$$

The provision of public goods in the bad state is $\tau - k$; *where* k *once again is the size of the bailout package.*

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1-\tau+k-x-\theta c_1)}{p_F\delta x} = \lambda_\beta$$

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta} - (1-\tau-x-\theta c_1)-k}{p_F\delta x} = \lambda_\beta$$

Our inequality above would then imply that

$$\frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta}^D - \left(1-\tau^D-x^D-\theta c_1^D\right)-k^D}{p_F\delta x^D} < \frac{(1-\theta)\widehat{\pi}_\beta c_{1\beta}^* - (1-\tau^*-x^*-\theta c_1^*)-k^*}{p_F\delta x^*}$$

We know from the good state feasibility constraint that:

$$(1-\theta)\,\widehat{\pi}_{\alpha} c_{1\alpha} = 1 - \tau - x - \theta c_1$$

$$\frac{(1-\theta)\widehat{\pi}_{\beta} c^{D}_{1\beta} - (1-\theta)\,\widehat{\pi}_{\alpha} c^{D}_{1\alpha} - k^{D}}{p_F \delta x^{D}} < \frac{(1-\theta)\widehat{\pi}_{\beta} c^{*}_{1\beta} - (1-\theta)\,\widehat{\pi}_{\alpha} c^{*}_{1\alpha} - k^{*}}{p_F \delta x^{*}}$$

From the inequalities that define the decentralized economy above we know, $c^{D}_{1\beta} > c^{*}_{1\beta}$, $c^{D}_{1\alpha} < c^{*}_{1\alpha}$, *and* $x^{D} < x^{*}$.

Therefore, for the inequality to hold, it must be true that

$$k^{D} > k^{*}$$

Proof of Proposition 11

Fragility Proof:

We have shown that $c^{D}_{1} < c^{*}_{1}$ *and* $c^{D}_{2\beta} > c^{*}_{2\beta}$ *which together imply that the economy is more fragile in the Planner's case than in the decentralized case.*

Stability Proof:

Starting with the stability condition around collateral liquidation.

$$\lambda_{\beta} < 1 - (1-\theta)(1-\widehat{\pi}_{\beta}) \left[\frac{R \cdot x + p_B\,(z - x\delta)}{x\delta p_B}\right]$$

The RHS of the inequality is decreasing in x. *Therefore, since the following holds true*

$$x^{D} < x^{*} \text{ and } \lambda^{D}_{\beta} < \lambda^{*}_{\beta},$$

Then, the inequality has more support in the decentralized case.